Optimistic and Topological Value Iteration for Simple Stochastic Games

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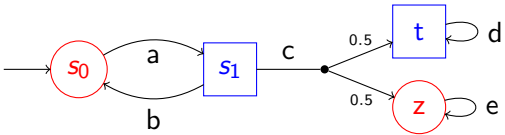
Alpine Verification Meeting, Frauenchiemsee, Germany

12.09.2022

Stochastic Game (SG)

Maximizer

Minimizer



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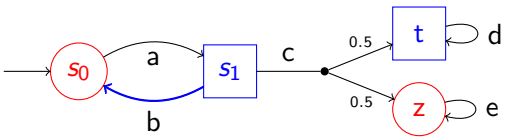

SGs with Reachability Objective – **Simple** Stochastic Games

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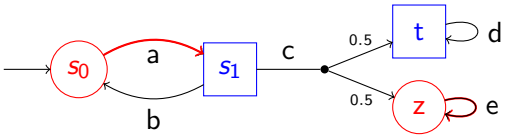

SGs with Reachability Objective – **Simple** Stochastic Games

Fix one player's strategy $\rightarrow$ **M**arkov **D**ecision **P**rocess (MDP)

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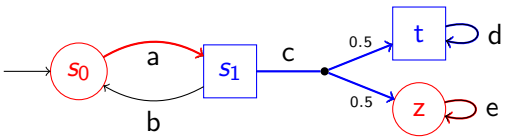
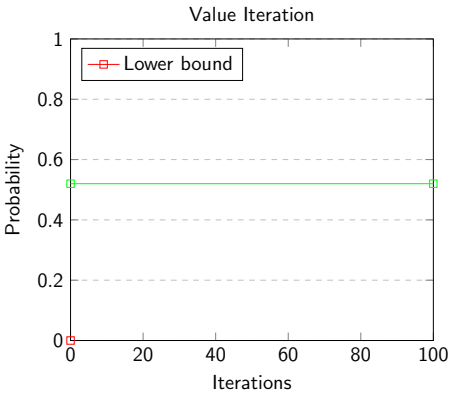


SGs with Reachability Objective – **Simple** Stochastic Games

Fix one player's strategy → **M**arkov **D**ecision **P**rocess (MDP)

Fix both players' strategies → **M**arkov **C**hain (MC)

Value Iteration (VI)



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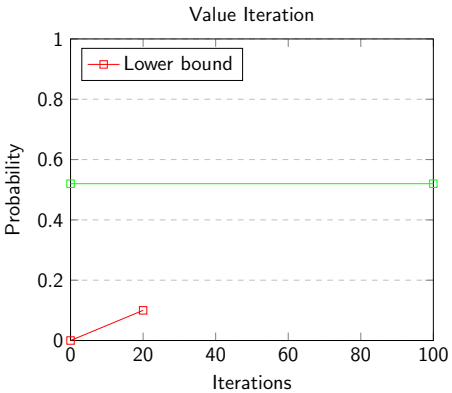

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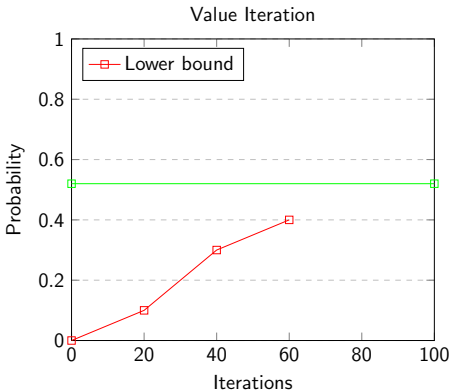
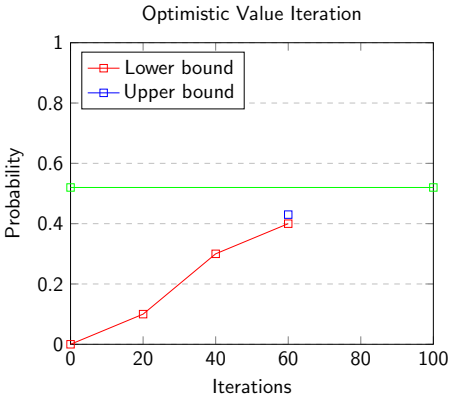


Termination: $\mathsf{L}_{i+1}(s) - \mathsf{L}_i(s) < \varepsilon$

Problem: no precision guarantee

Optimistic Value Iteration (OVI)

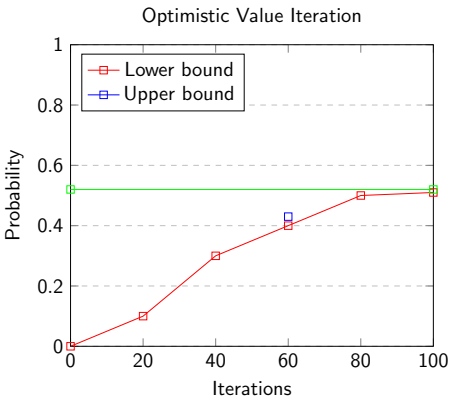


Optimistic with naive VI

Idea: **guess** U and **verify**

Extended OVI from MDPs to SGs

Optimistic Value Iteration (OVI)



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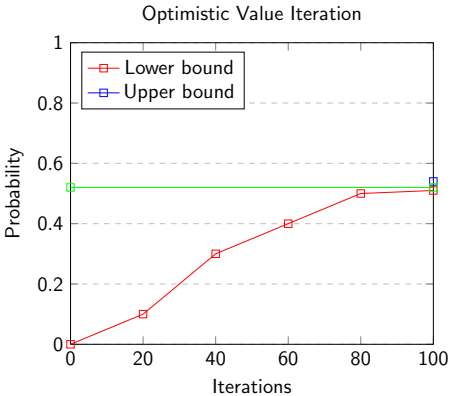

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Scaling up Value Iteration

From models with a few thousand states to models with a few million of states

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Idea: break the problem into smaller problems and solve piece by piece

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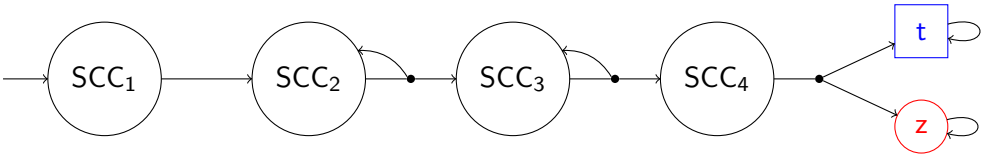
Exploit the graph structure

Strongly **C**onnected **C**omponents (SCCs) of an SG form a **D**irected **A**cyclic **G**raph

Topological VI (TVI)



Solve SCCs in bottom-up of their topological ordering

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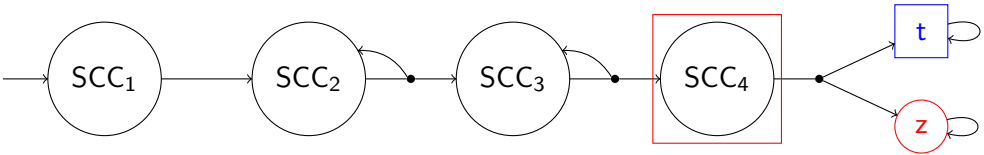

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Approximate [L, U] for SCCs

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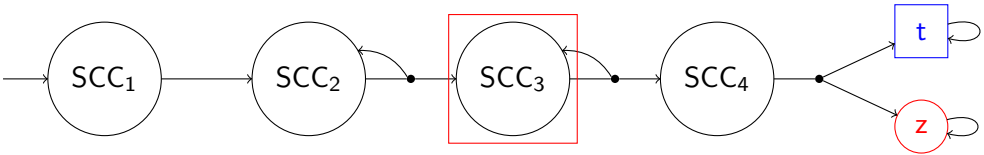

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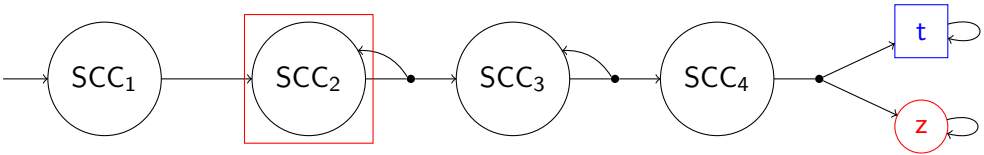

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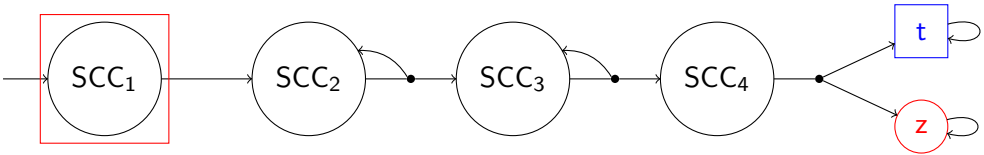

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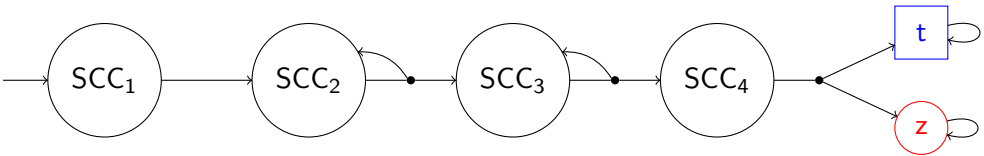
Problem

Error aggregate through SCCs

- long chain of SCCs: large aggregated error
- leads to **non-termination**

Precise Topological Value Iteration



Solves the problem of non-termination for topological VI

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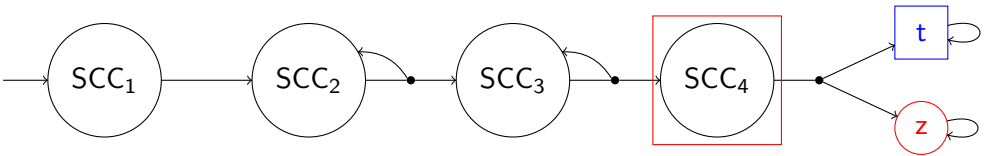

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Export the (near-optimal) strategies for approximate values

Compute the precise values by solving the underlying MC

Verify if the strategies are indeed optimal

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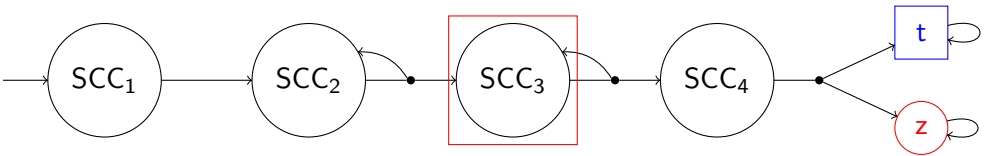

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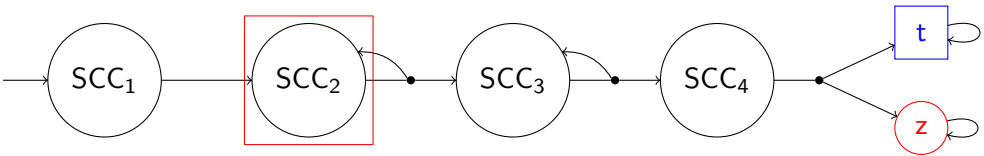

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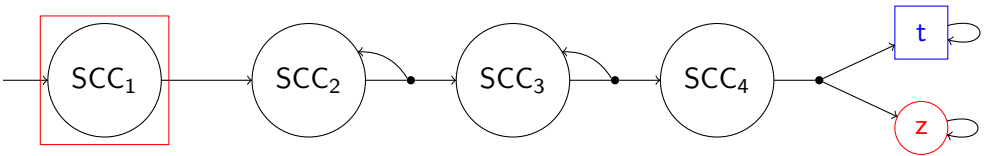

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Take Away

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- extension to SGs
- **guess** U and **verify**

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Optimistic Value Iteration

- extension to SGs
- **guess** U and **verify**

Precise Topological Value Iteration

- fixed non-termination issue of topological VI
- export strategies as a **guess**, and **verify** if these are indeed optimal

Thank you!