Transition Power Abstraction

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September 12, 2022
14th Alpine Verification Meeting

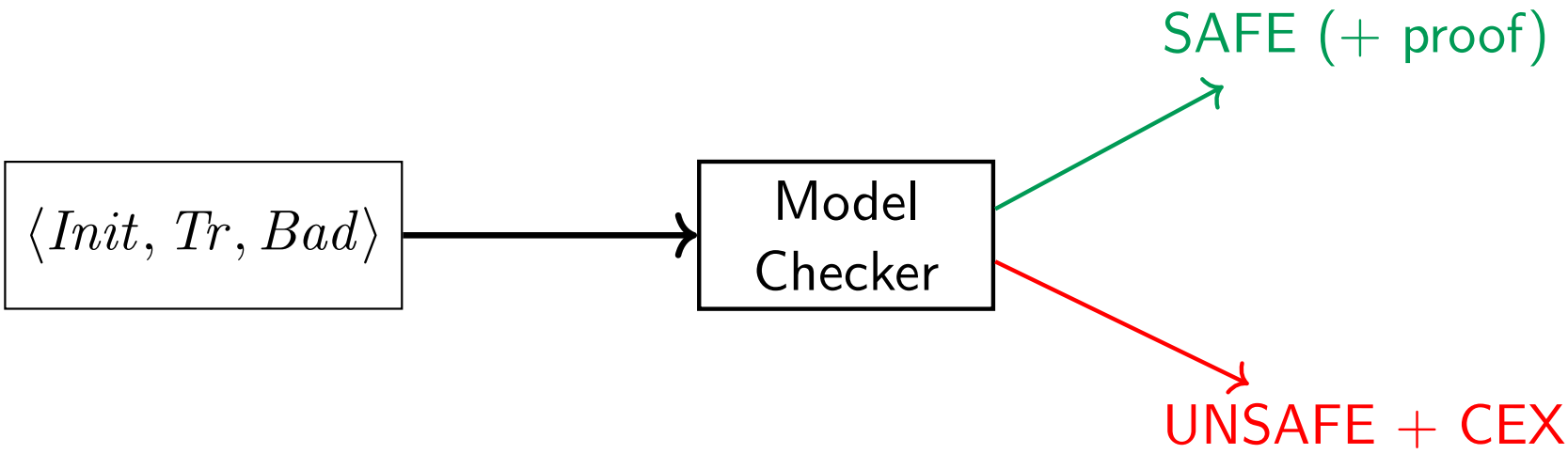
Many thanks to my colleagues!

Antti Hyvärinen

Grigory Fedyukovich

Natasha Sharygina

Context



SMT-based model checking of transition systems (with interpolants)

Motivation

```
x = 0;
y = N;
while(x < 2N)
{
   x = x + 1;
   if(x > N)
      y = y + 1;
}
assert(y != 2N);
```

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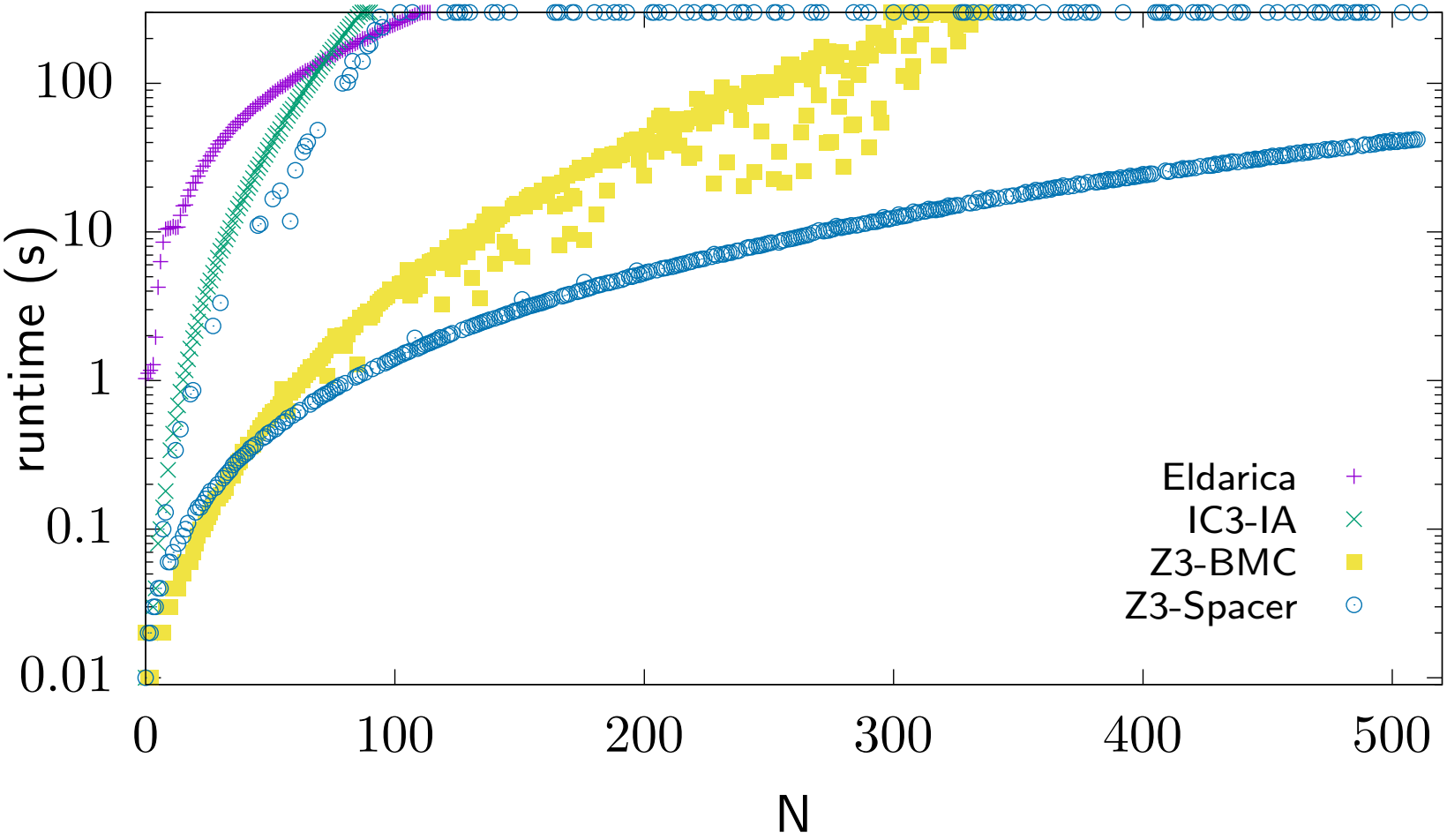
assert(y != 2N);
```

$$Init \equiv x = 0 \wedge y = N$$

$$Tr \equiv x < 2N \wedge x' = x + 1 \wedge y' = ite(x < 2N, y, y + 1)$$

$$Bad \equiv x \geq 2N \wedge y = 2N$$



Performance of state-of-the-art tools for increasing counterexample length

Problem: Slow progress in search for longer counterexamples

Solution: Summarize multiple steps

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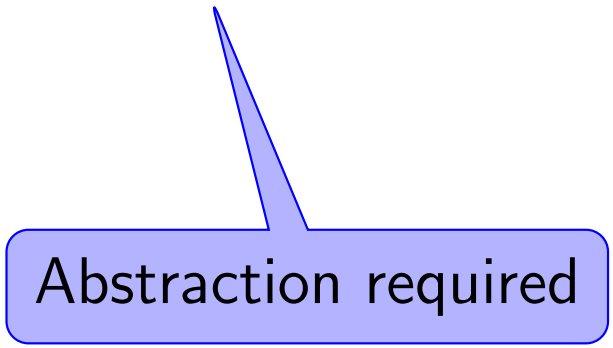

Transition power abstraction sequence

TPA sequence $TPA^0, TPA^1, \ldots, TPA^n, \ldots$

$TPA^n(x, x')$

- overapproximates reachability up to 2^n steps of Tr
- quantifier-free (only 2 copies of state variables)

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- How to construct such sequence?
- How to build a model checking algorithm around it?

Main algorithm

Global: $TPA^0, TPA^1, \ldots$ lazy-initialized to $true$

`CheckSafety`$(Init, Tr, Bad)$

1: $TPA^0 = Id \vee Tr$
2: $n = 0$
3: **while** $true$
4: **if** `IsReachable`$(Init, Bad, n)$
5: **return UNSAFE**
6: $n = n + 1$

Main algorithm

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up to 2^{n+1} steps of Tr

IsReachable procedure

`IsReachable`($Source, Target, n$)

1: $res = Sat?[Source(x) \wedge TPA^n(x, x') \wedge TPA^n(x', x'') \wedge Target(x'')]$
2: **if** $res == UNSAT$
3: $\wedge\ Target(x'')$
4:
5: **return false**
6: **else** // $res == SAT$
7: **if** $n == 0$
8: **return true**
9: $Intermediate =$ `ExtractIntermediate()`
10: **if not** `IsReachable`($Source, Intermediate, n-1$)
11: goto 1
12: **if not** `IsReachable`($Intermediate, Target, n-1$)
13: goto 1
14: **return true**

Is $Target$ reachable from $Source$ in $\leq 2^{n+1}$ steps

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Abstract path exists?

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3: $I = Itp(TPA^n(x, x') \wedge TPA^n(x', x''), Source(x) \wedge Target(x''))$
4: $TPA^{n+1} = TPA^{n+1} \wedge I$
5: **return false**
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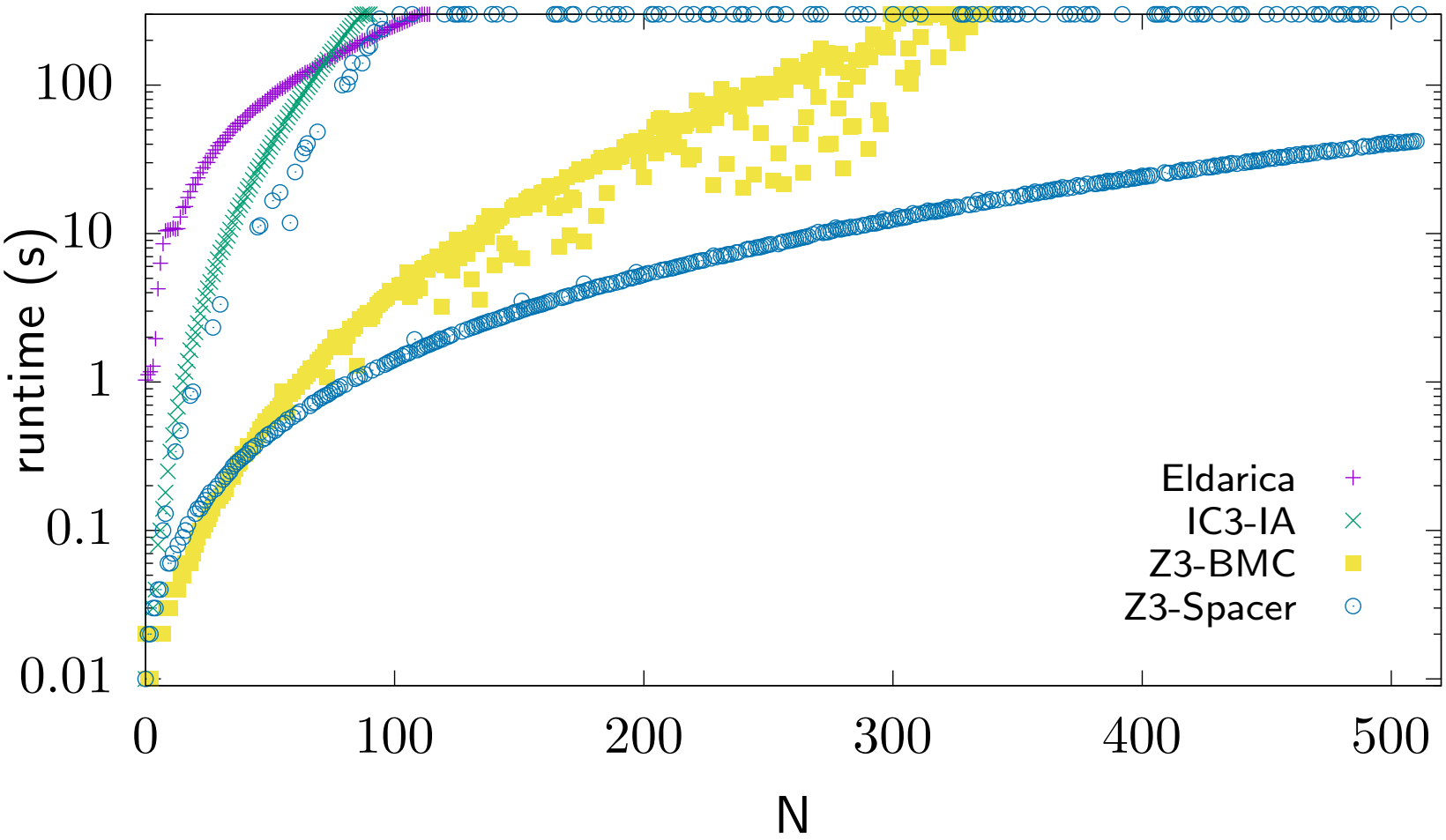
Experiments



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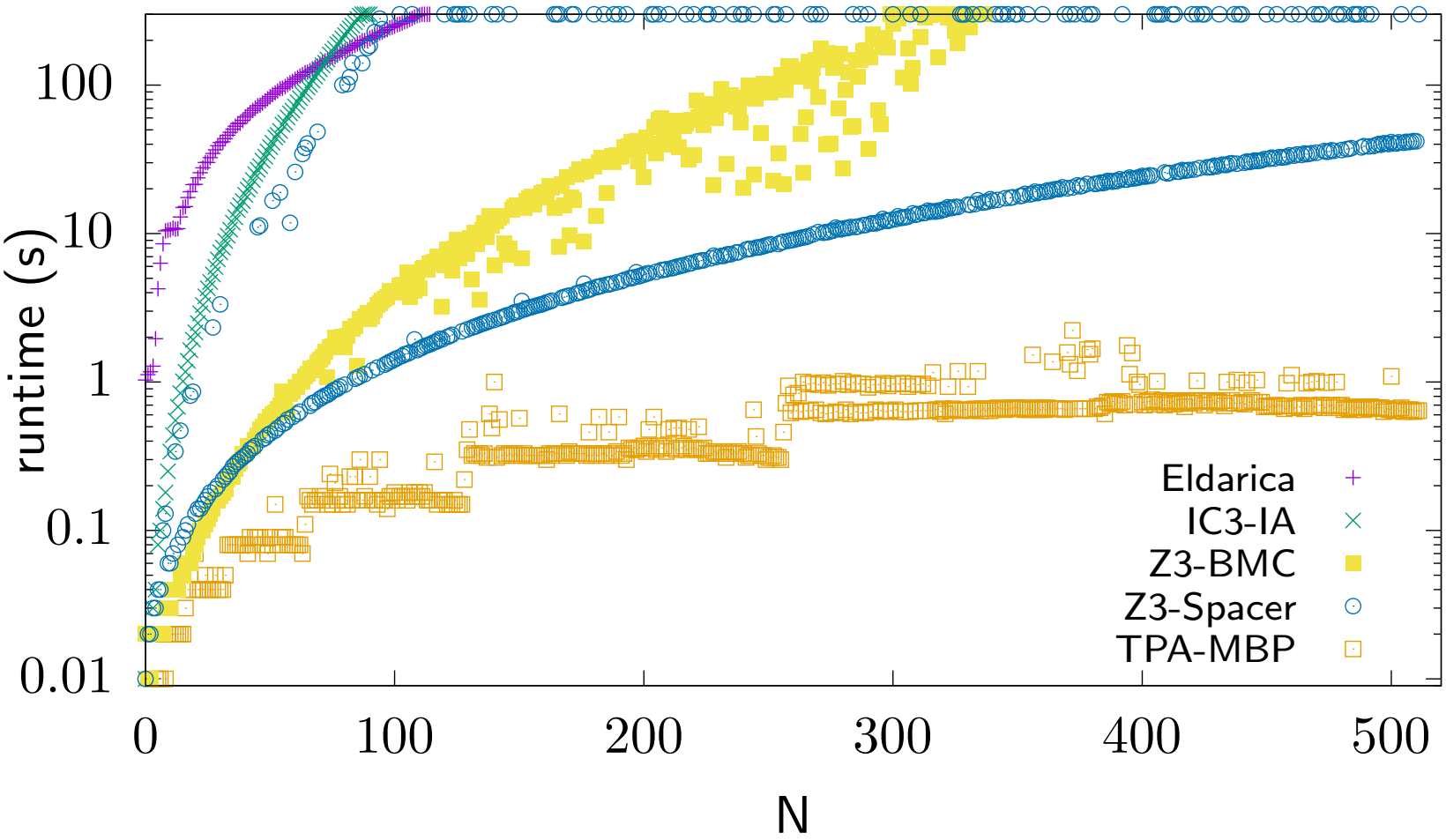

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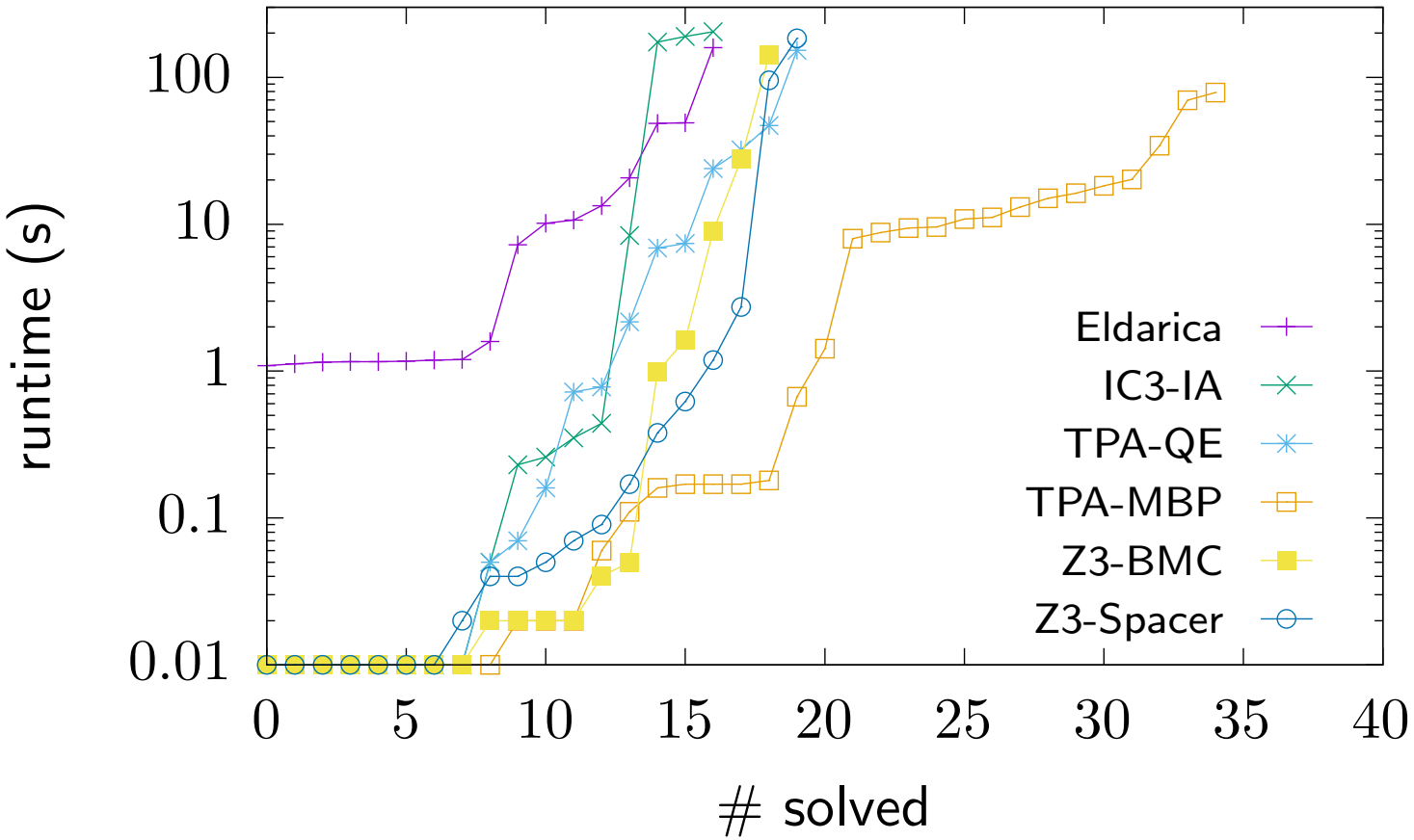

Performance on 54 unsafe multi-phase loops

Proving safety

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- Elements of TPA as candidates for safe inductive transition invariants

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- Elements of TPA as candidates for safe inductive transition invariants

- Come to FMCAD'22 in Trento!

Summary

- *New* algorithm for safety properties of transition systems
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- Transition invariants [TACAS'22 + FMCAD'22]

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- *New* algorithm for safety properties of transition systems
  - Transition power abstraction sequence
- Deep counterexample detection [TACAS'22]
- Transition invariants [TACAS'22 + FMCAD'22]
- Implemented in Horn solver Golem

  `https://github.com/usi-verification-and-security/golem`

Thank you!

Questions?