Inclusion Checking for ω-VPL

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Pierre Ganty

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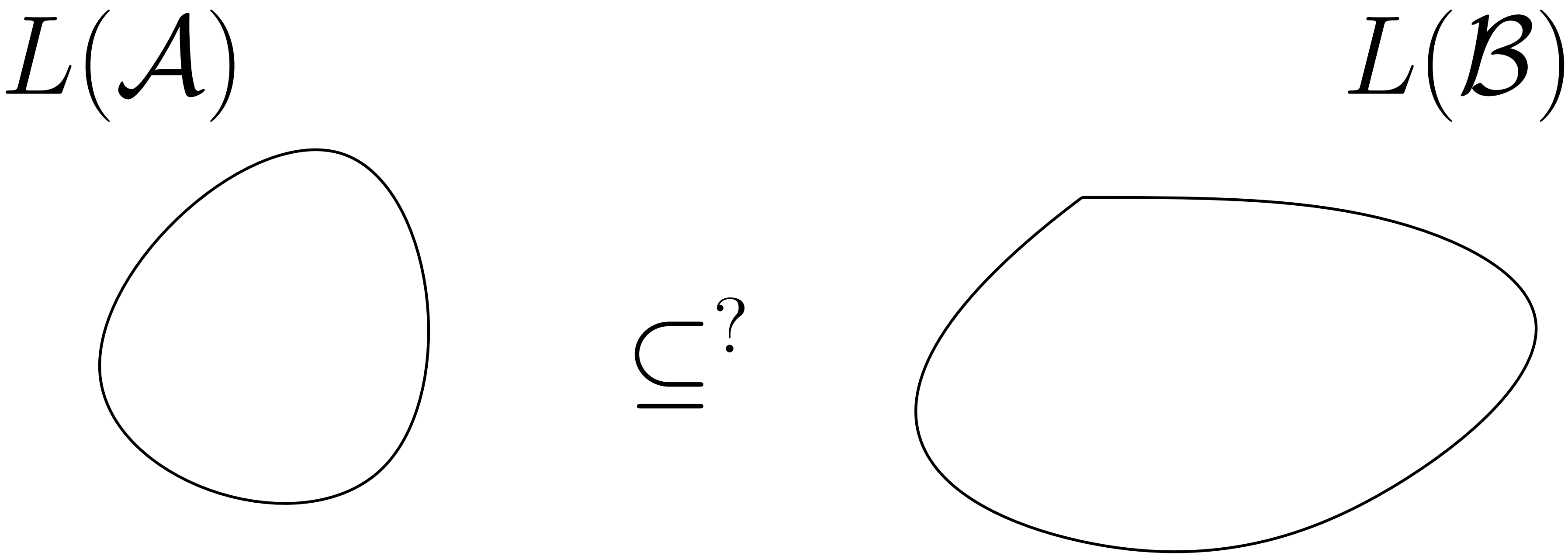
IMDEA Software Institute

Inclusion Checking for ω-VPA



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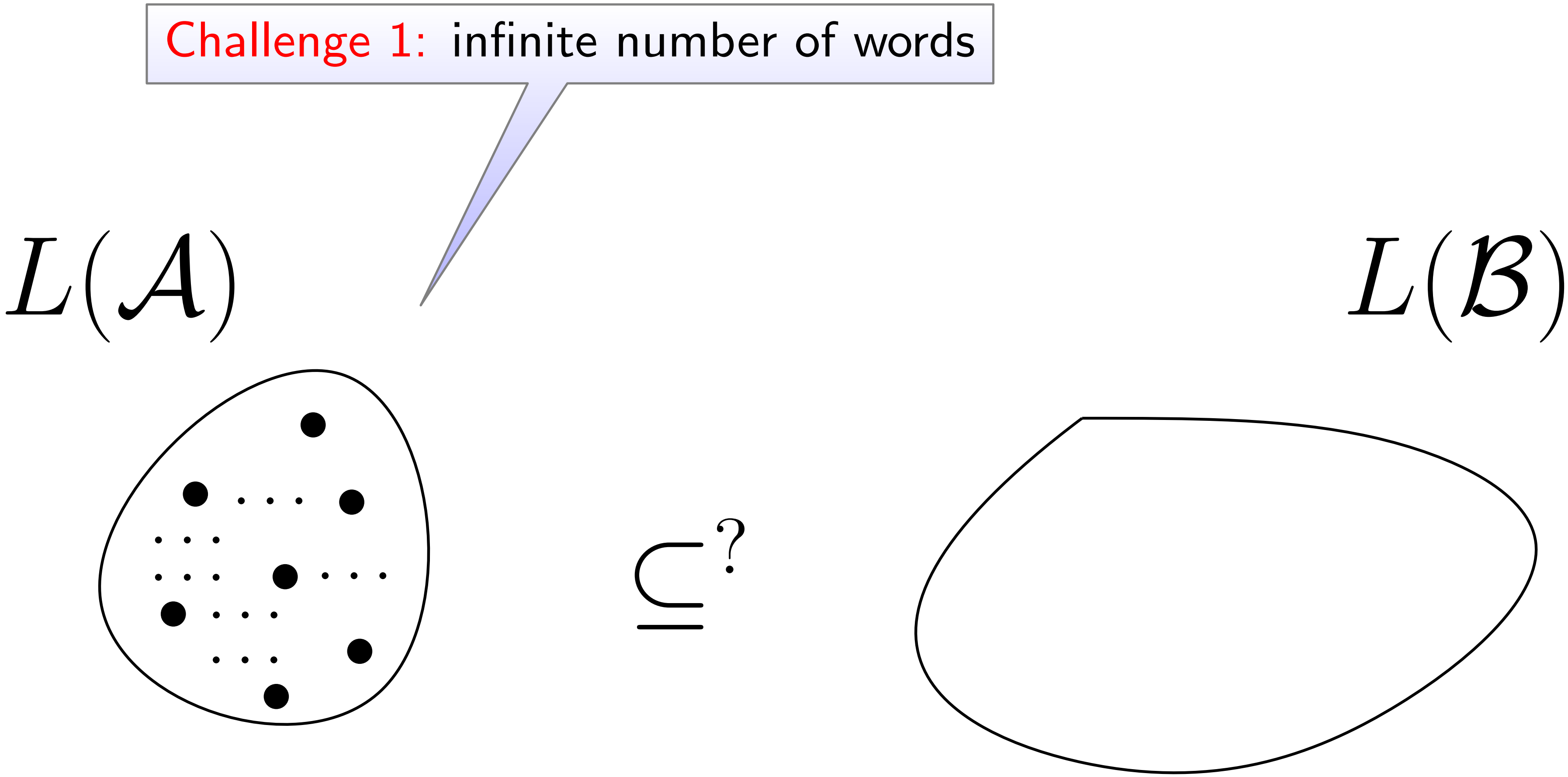

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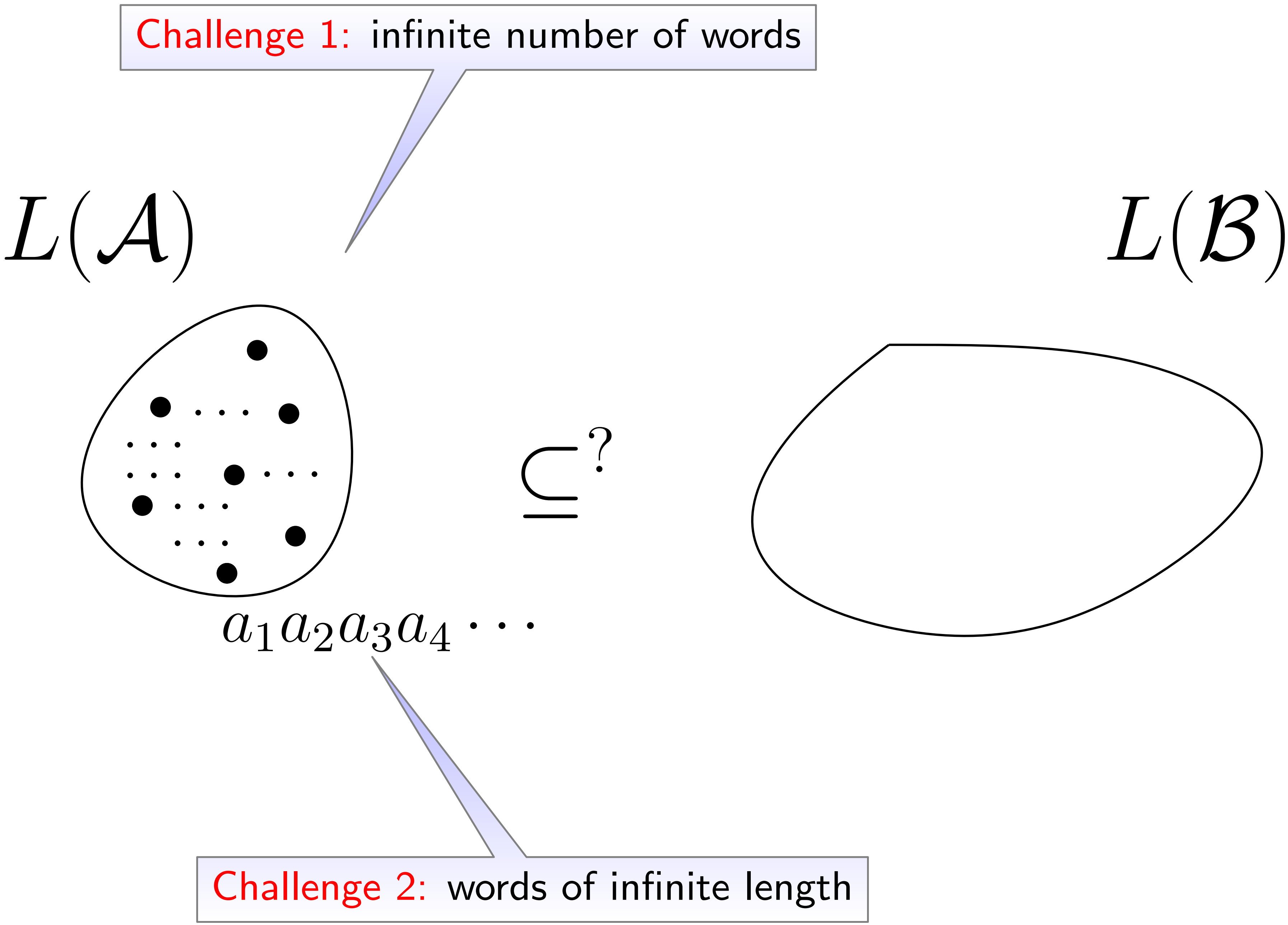
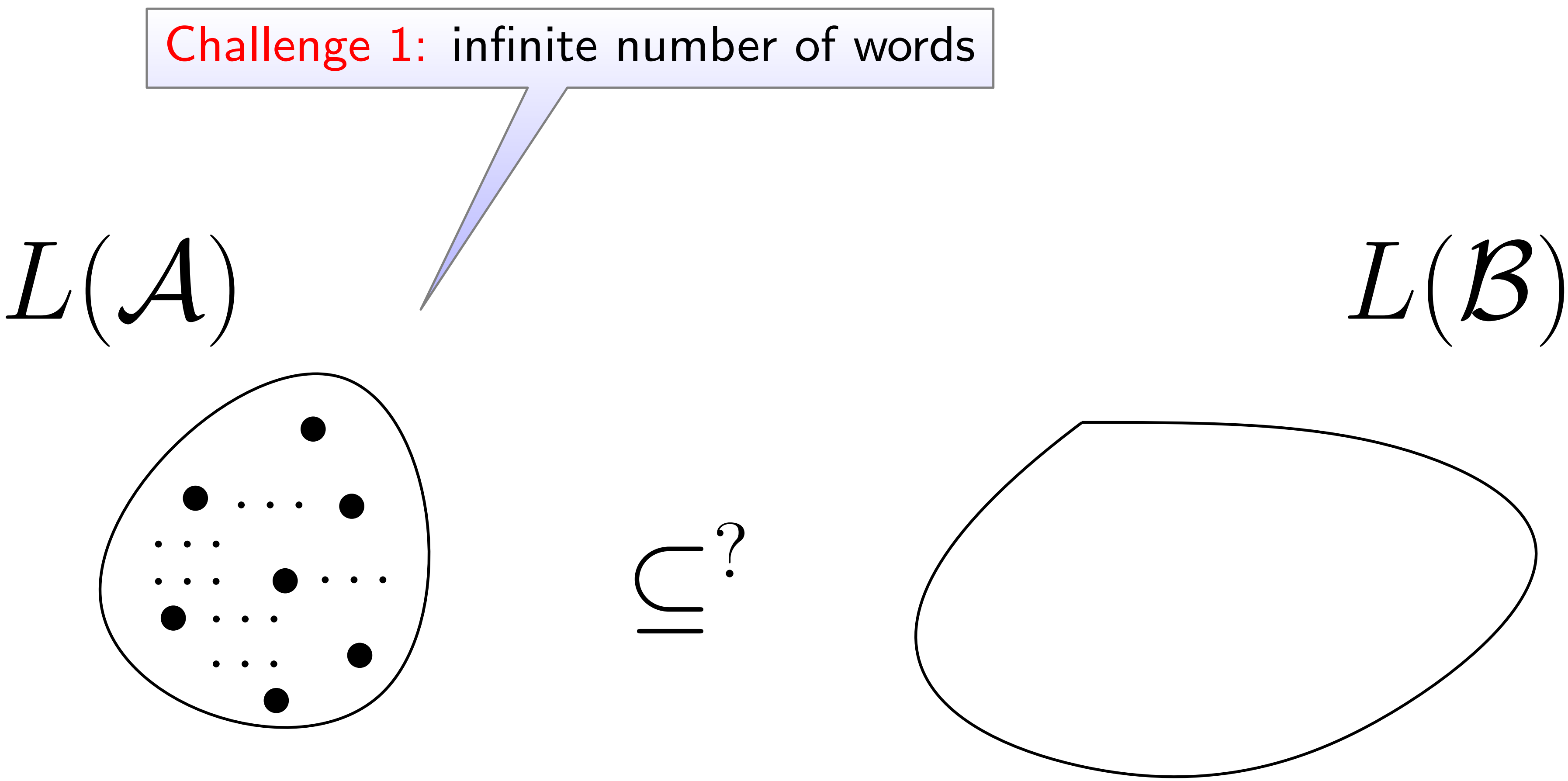


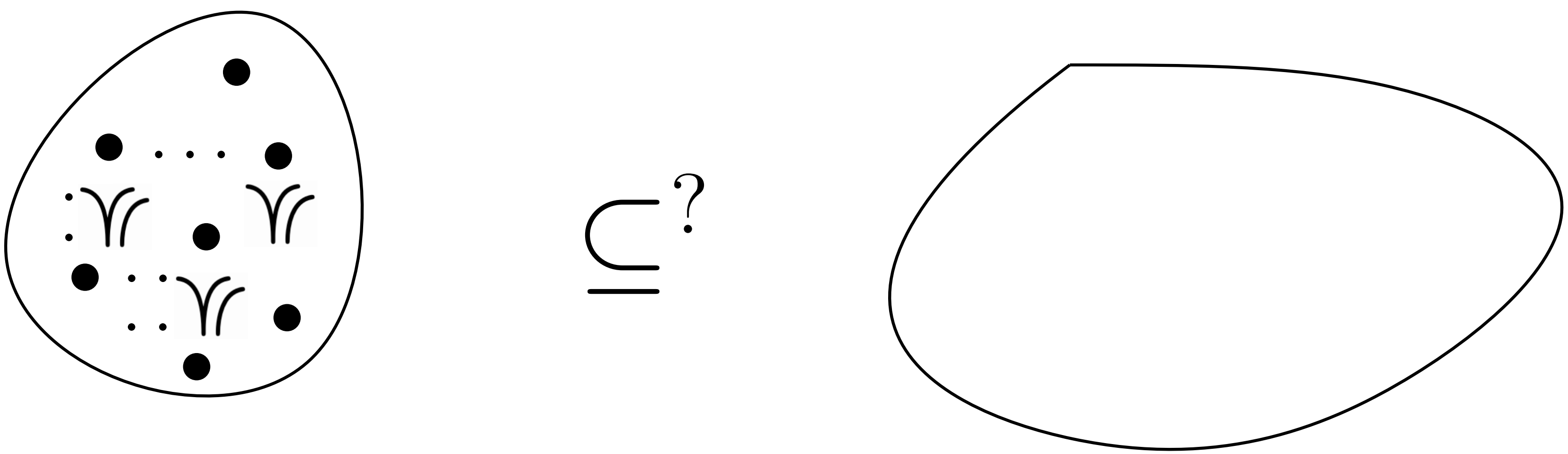
Quasiorder-based Inclusion Checking of ω-VPA



Quasiorder-based Inclusion Checking of ω-VPA

Challenge 1: infinite number of words

$L(\mathcal{A})$ $\subseteq^?$ $L(\mathcal{B})$



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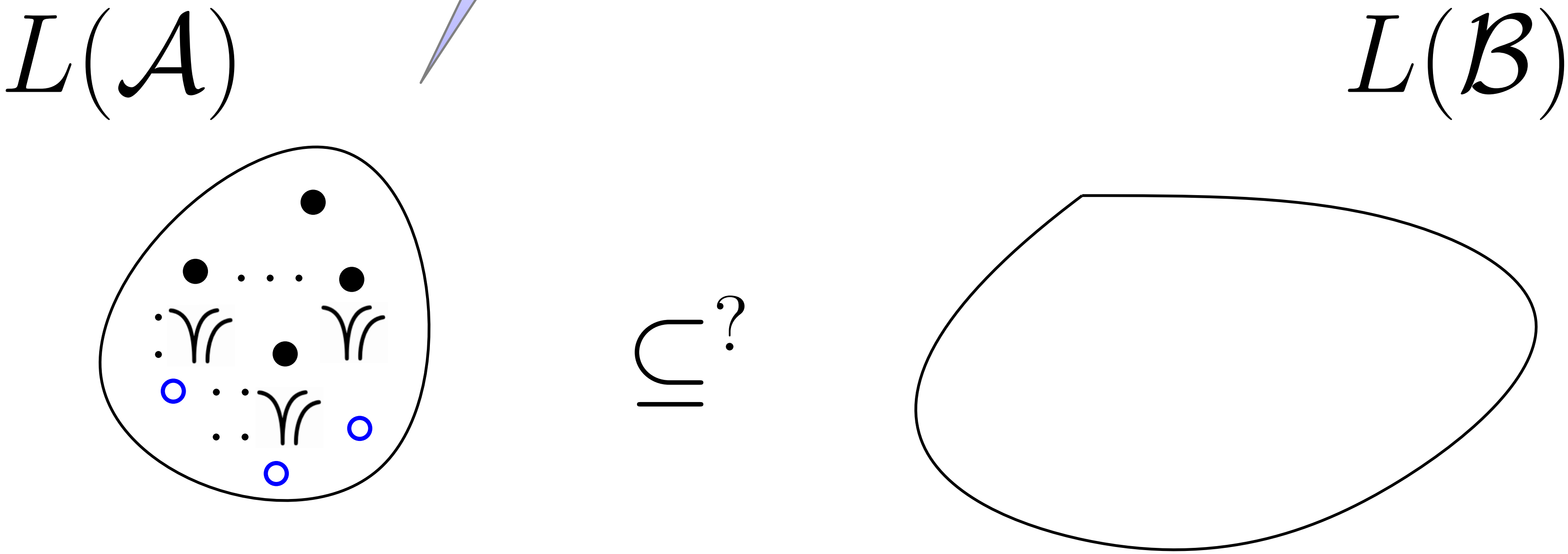

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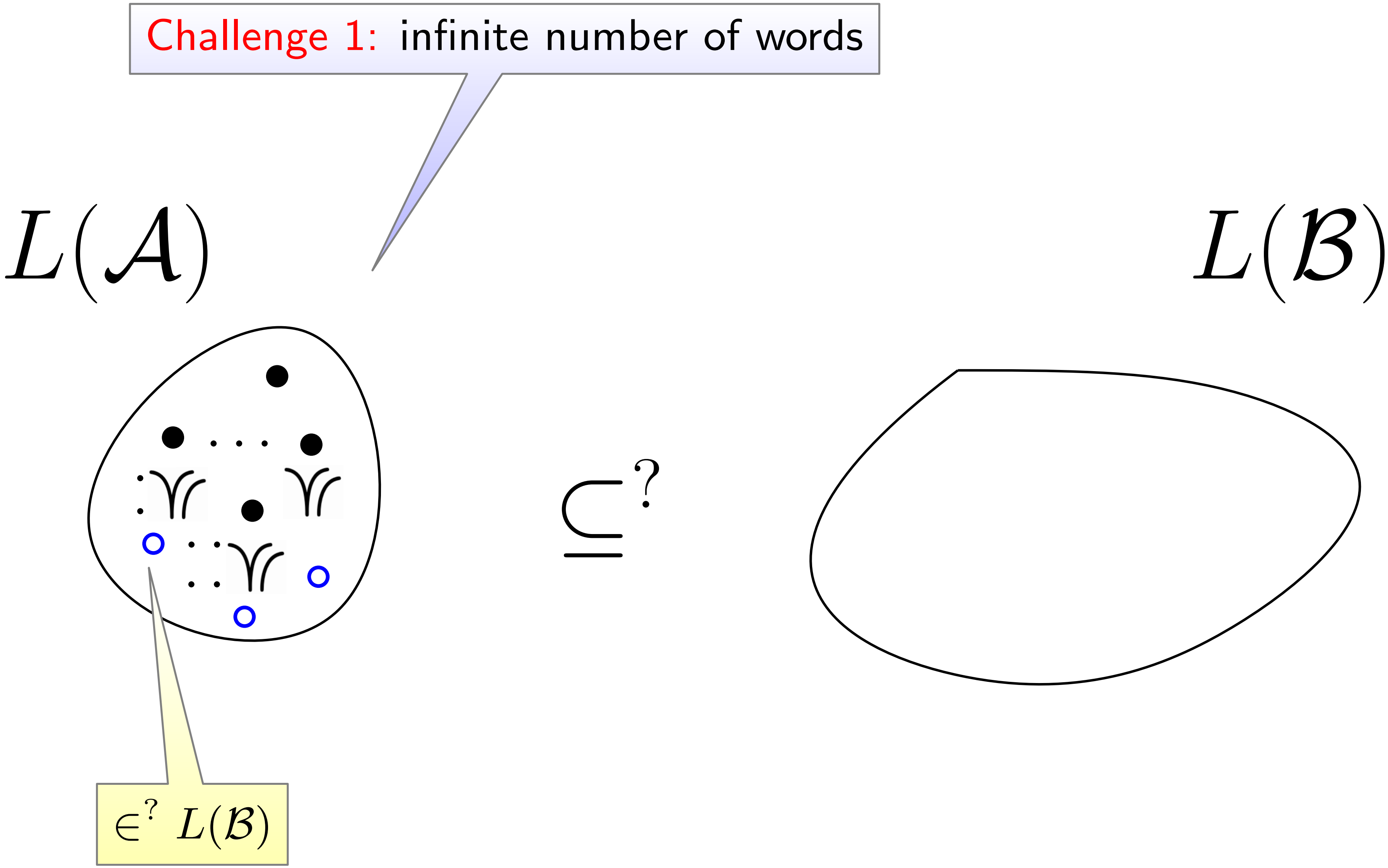

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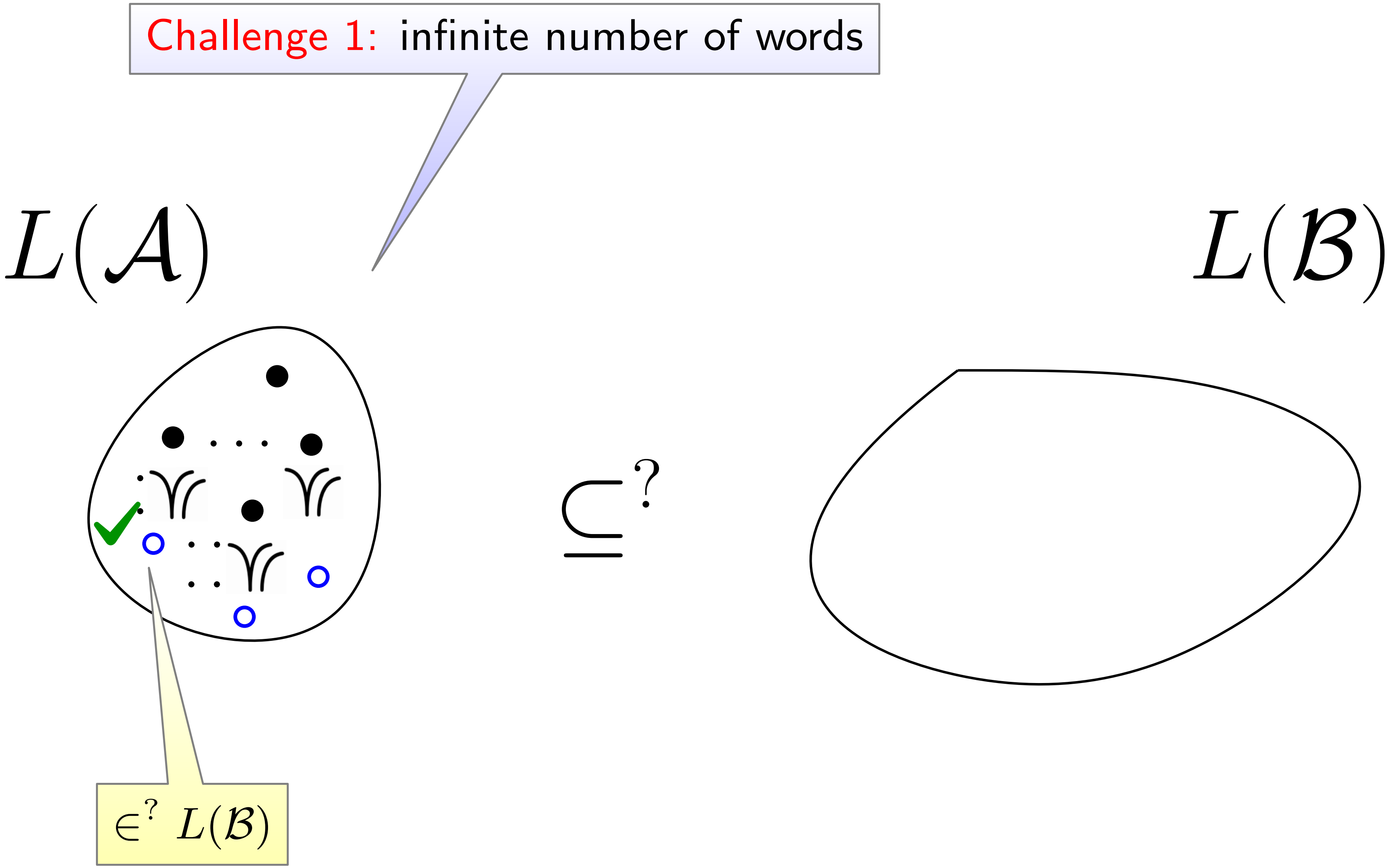

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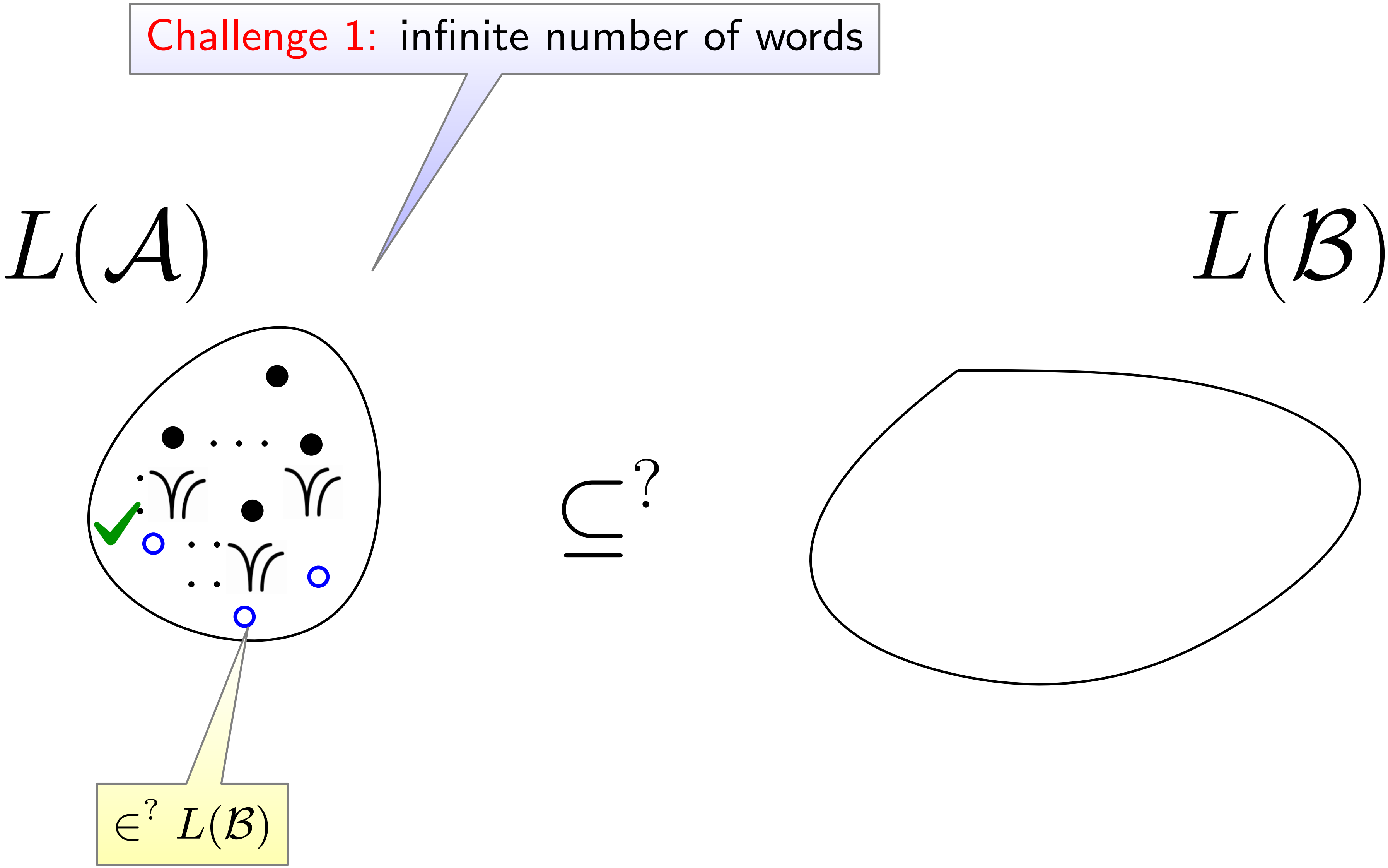

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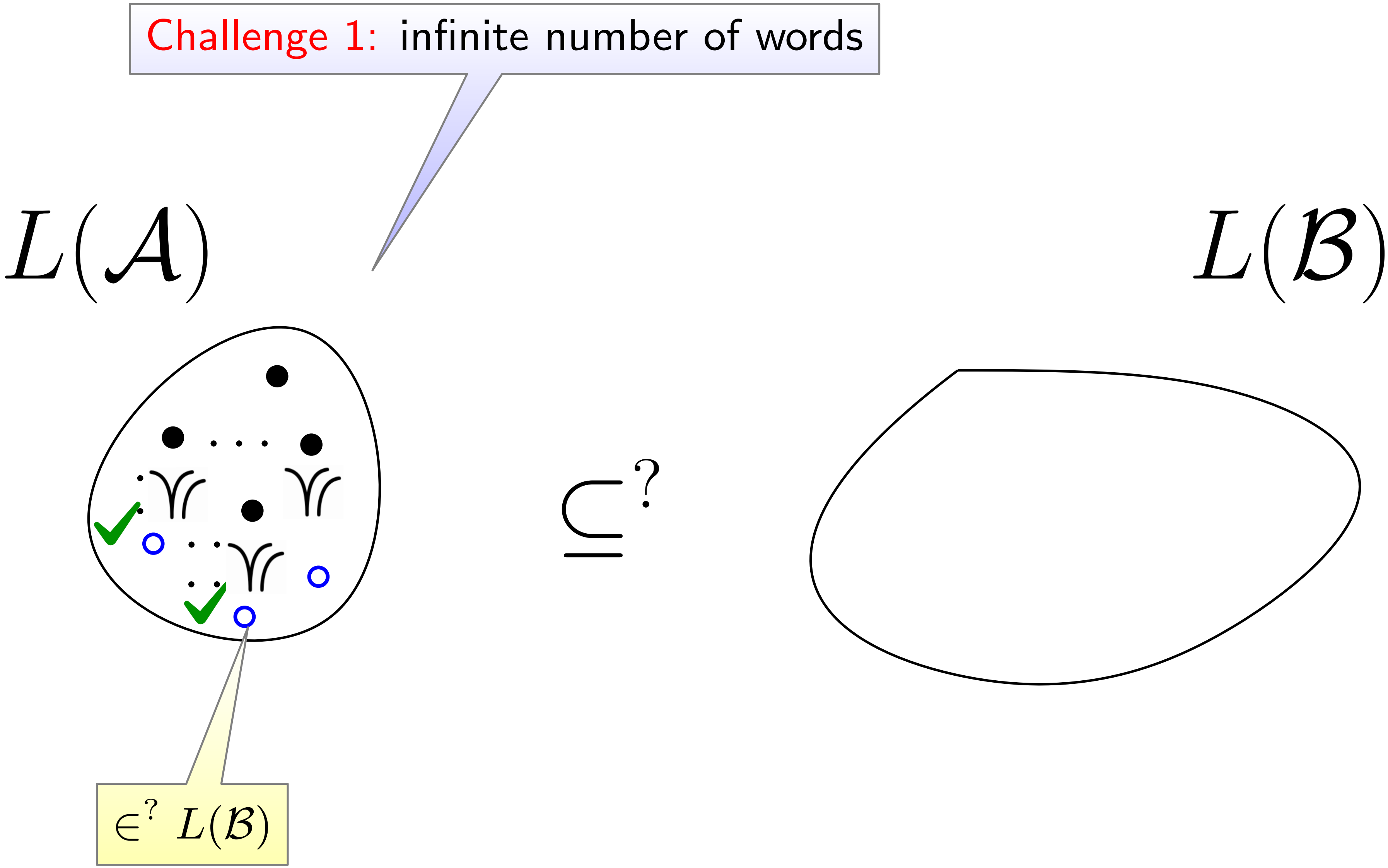

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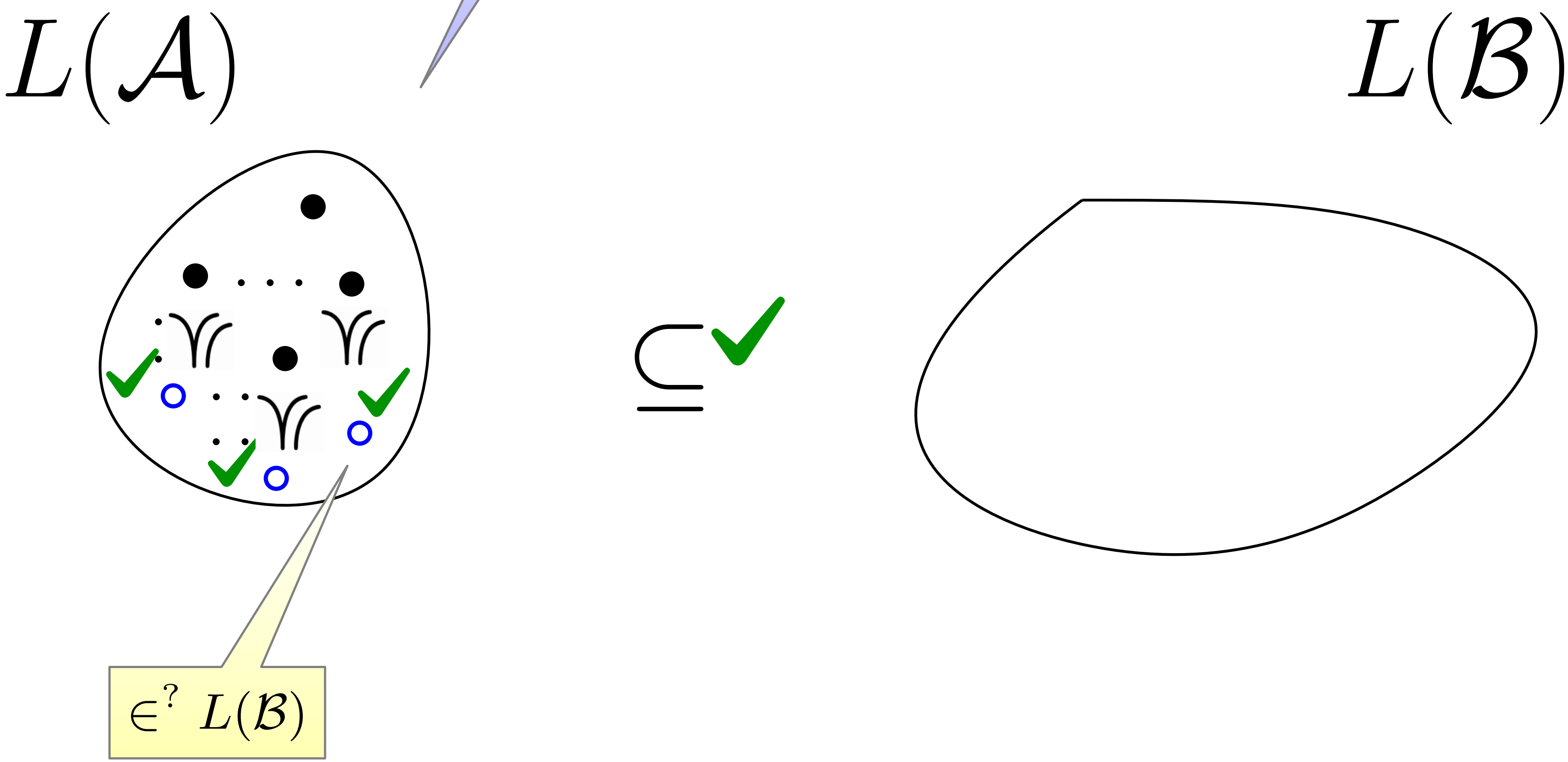

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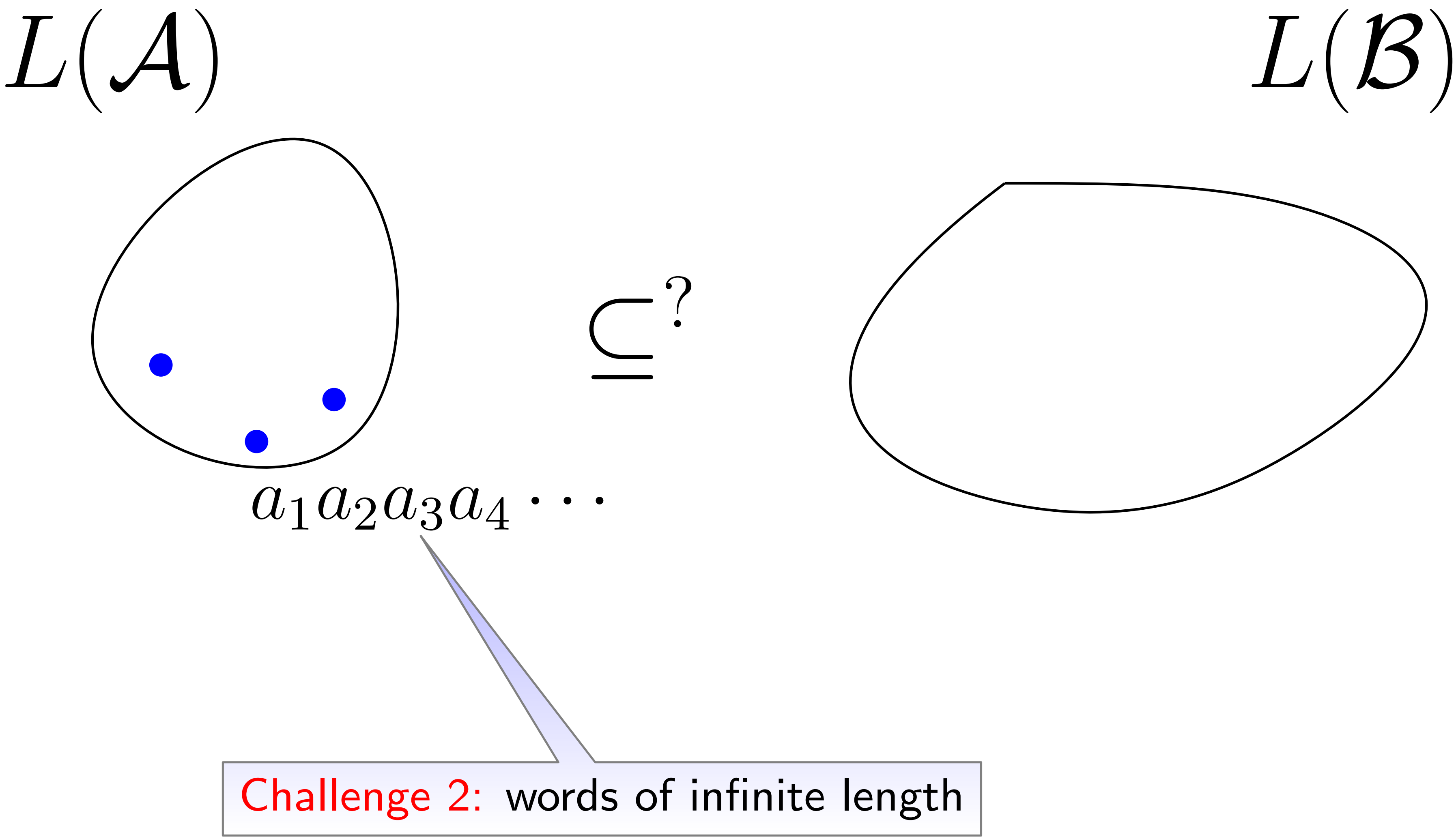
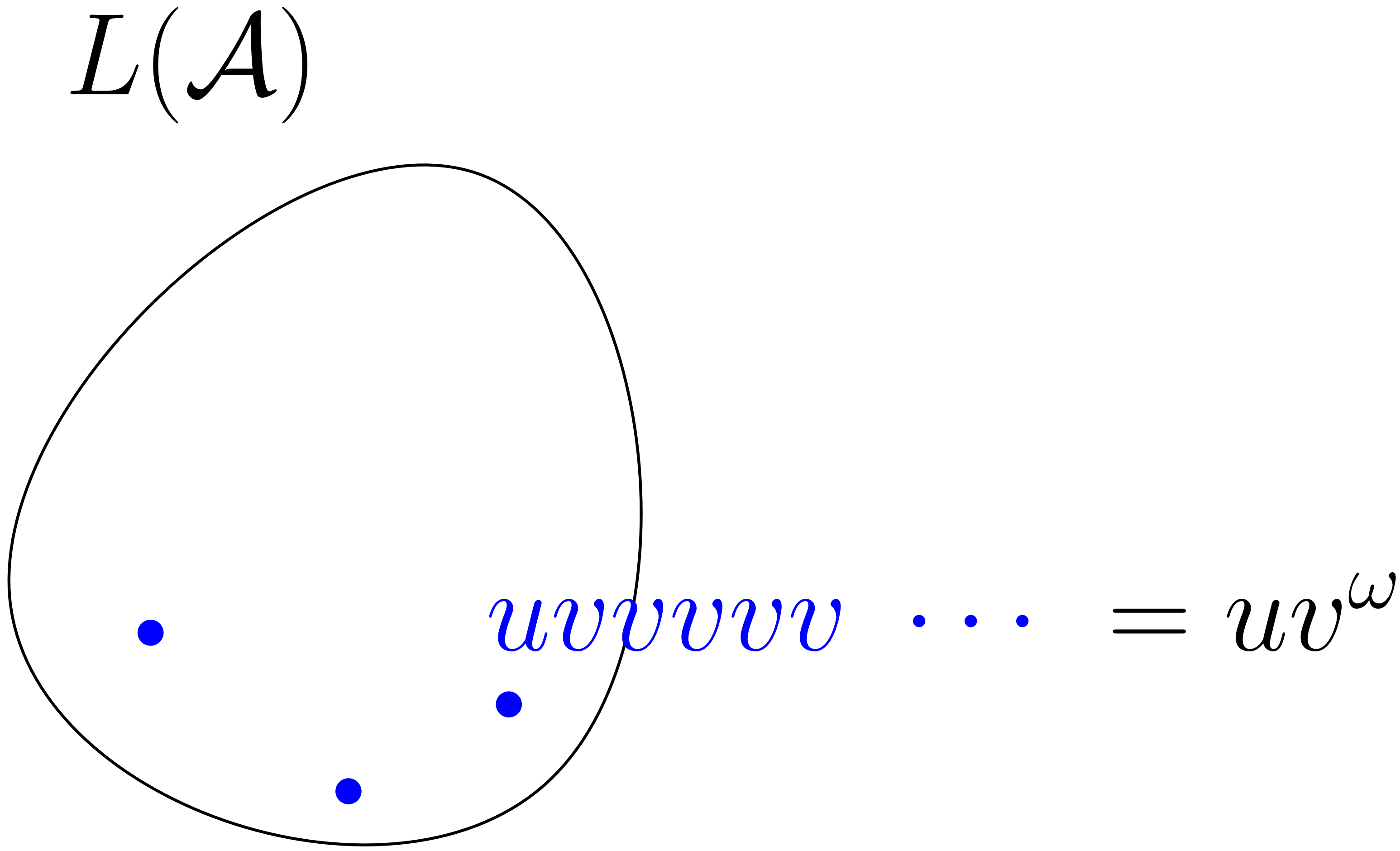


Ultimately Periodic Words



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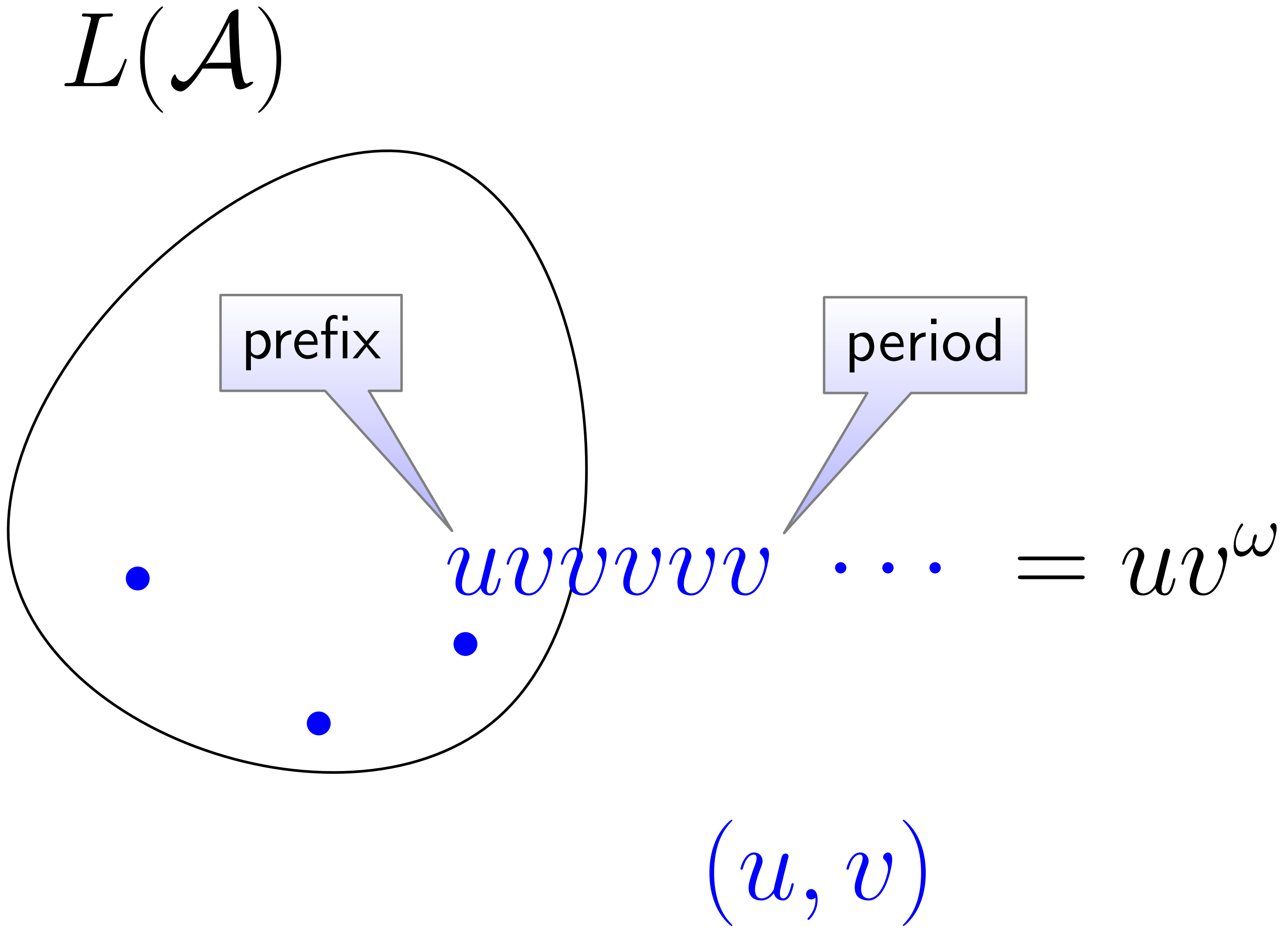

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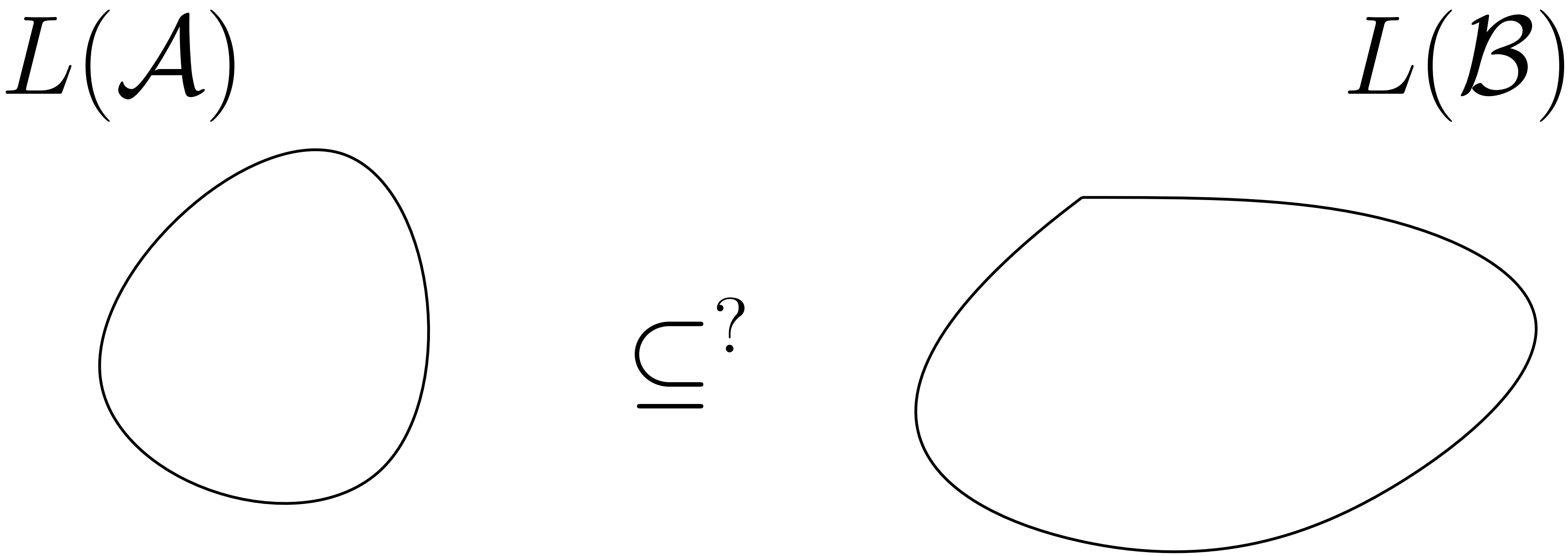
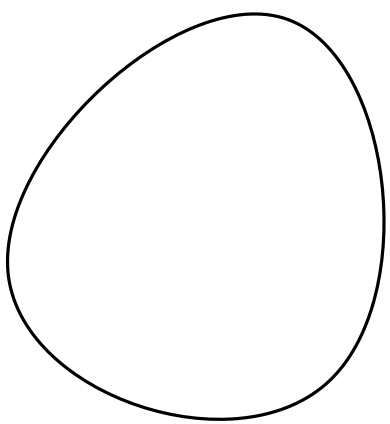


Quasiorder-based Inclusion Checking of ω-VPA

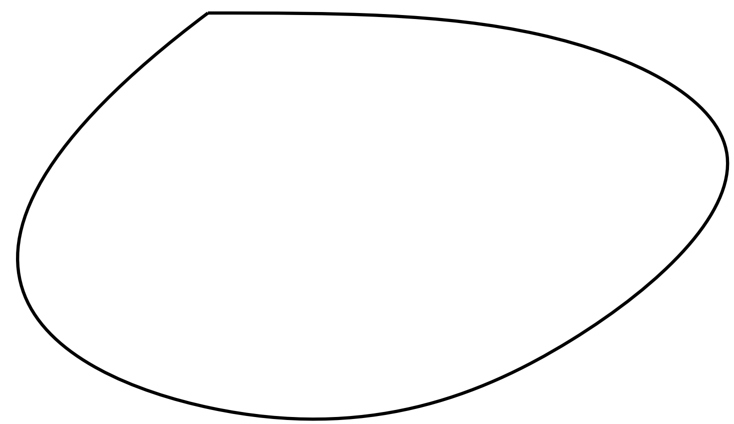
prefix period

$$\leq, \preceq$$

$L(\mathcal{A})$ $L(\mathcal{B})$



$$\subseteq^?$$



Quasiorder-based Inclusion Checking of ω-VPA

prefix period

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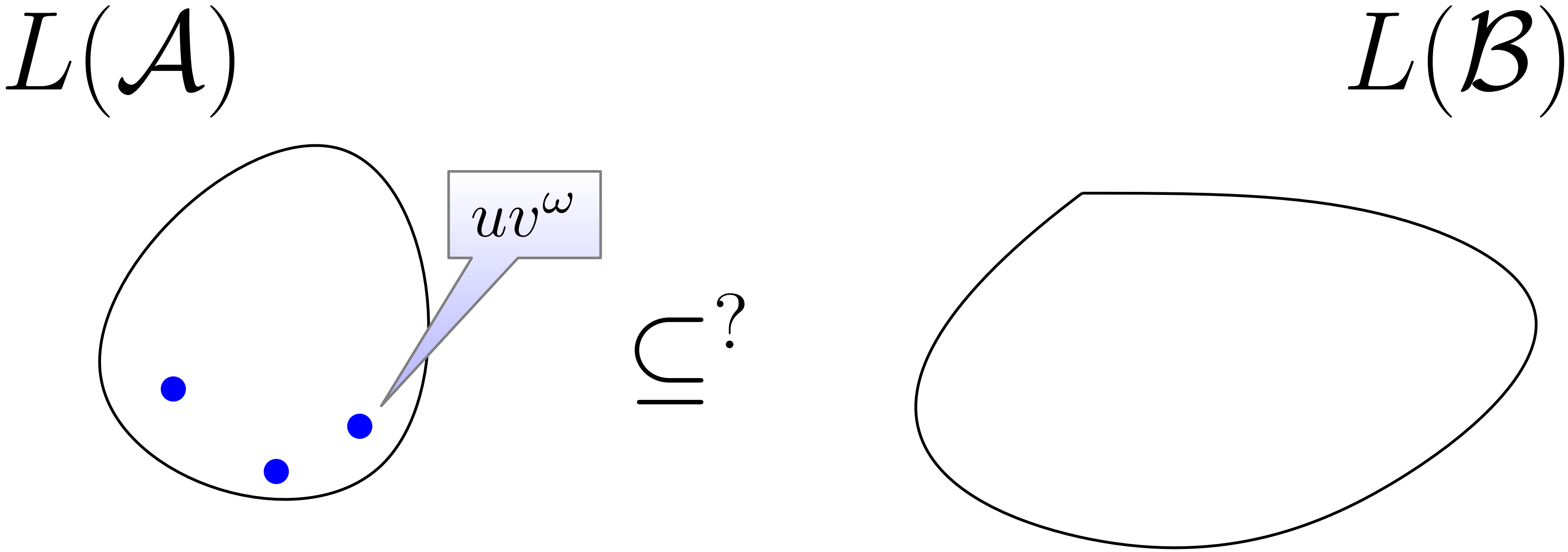
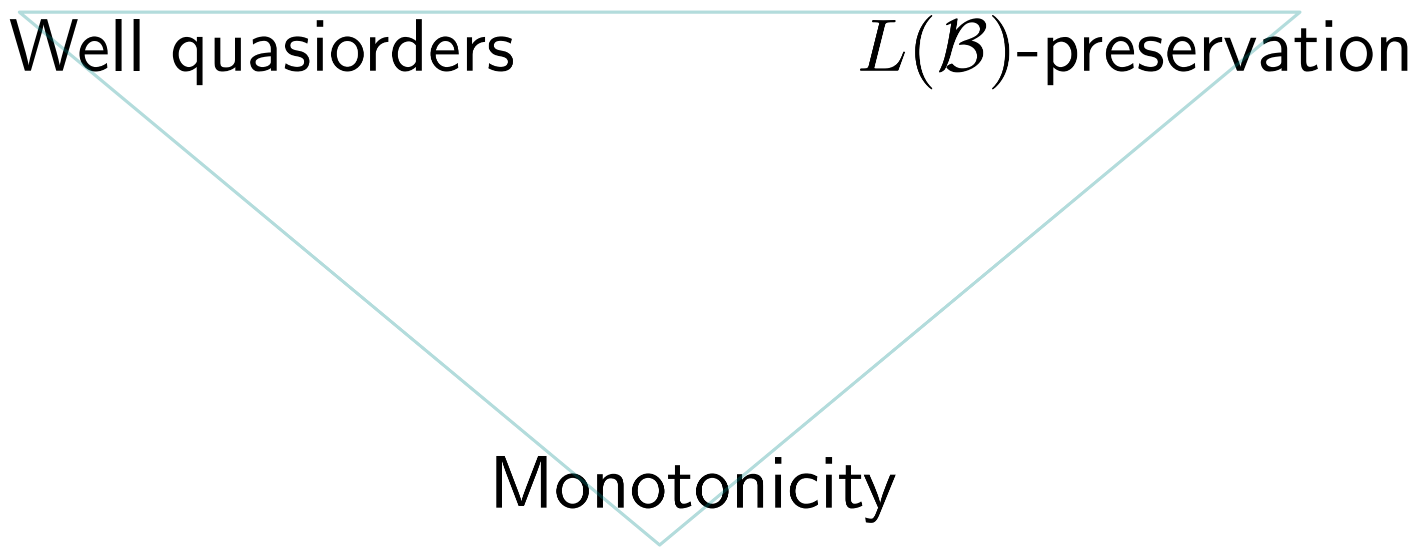


$$L(\mathcal{A}) \subseteq L(\mathcal{B}) \iff \forall (u,v) \in S_{finite}, uv^\omega \in L(\mathcal{B})$$

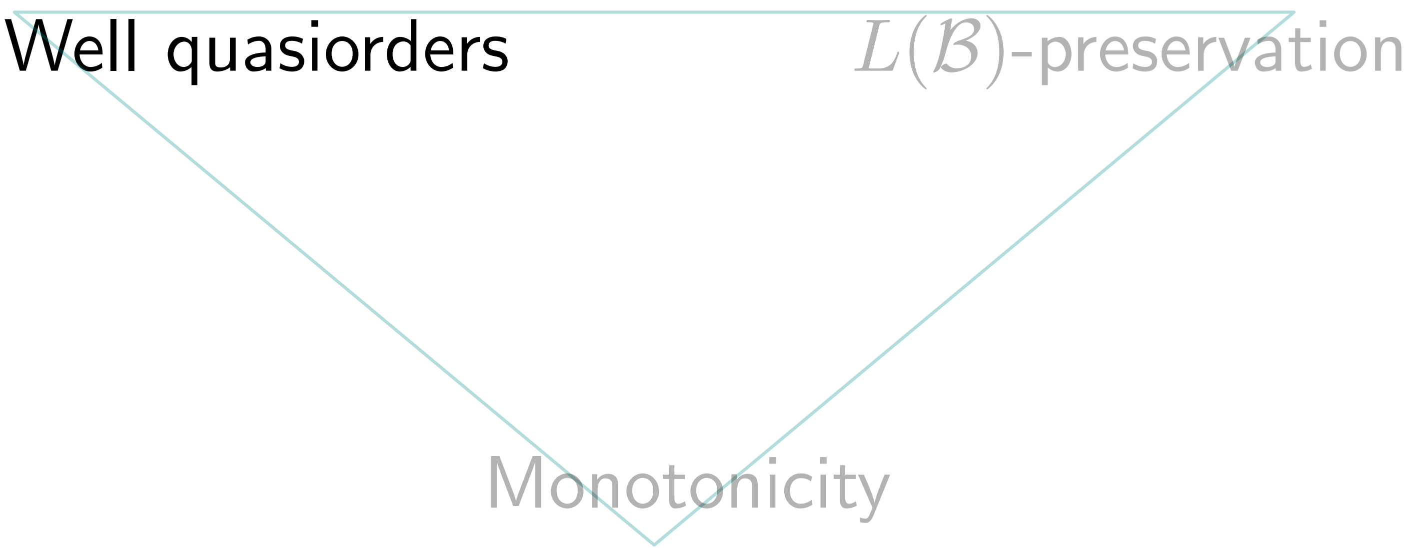
Requirements

$$\leq, \preceq$$



Requirements

$$\leq, \preceq$$



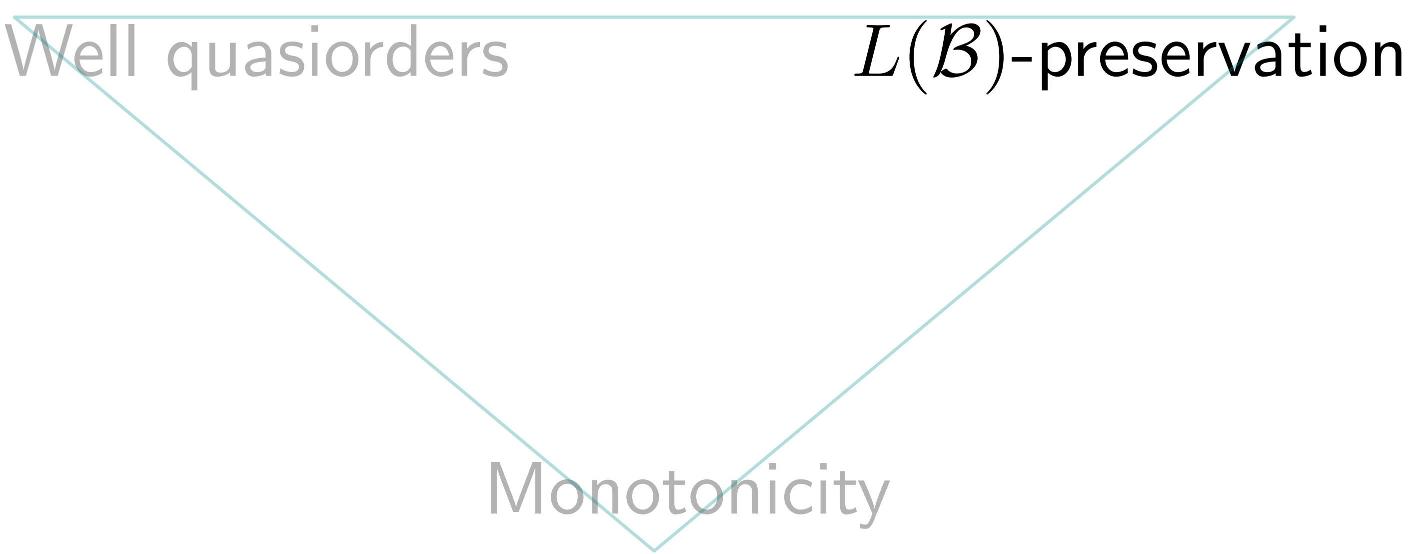
Well quasiorders

if $\leq$ is a well quasiorder then

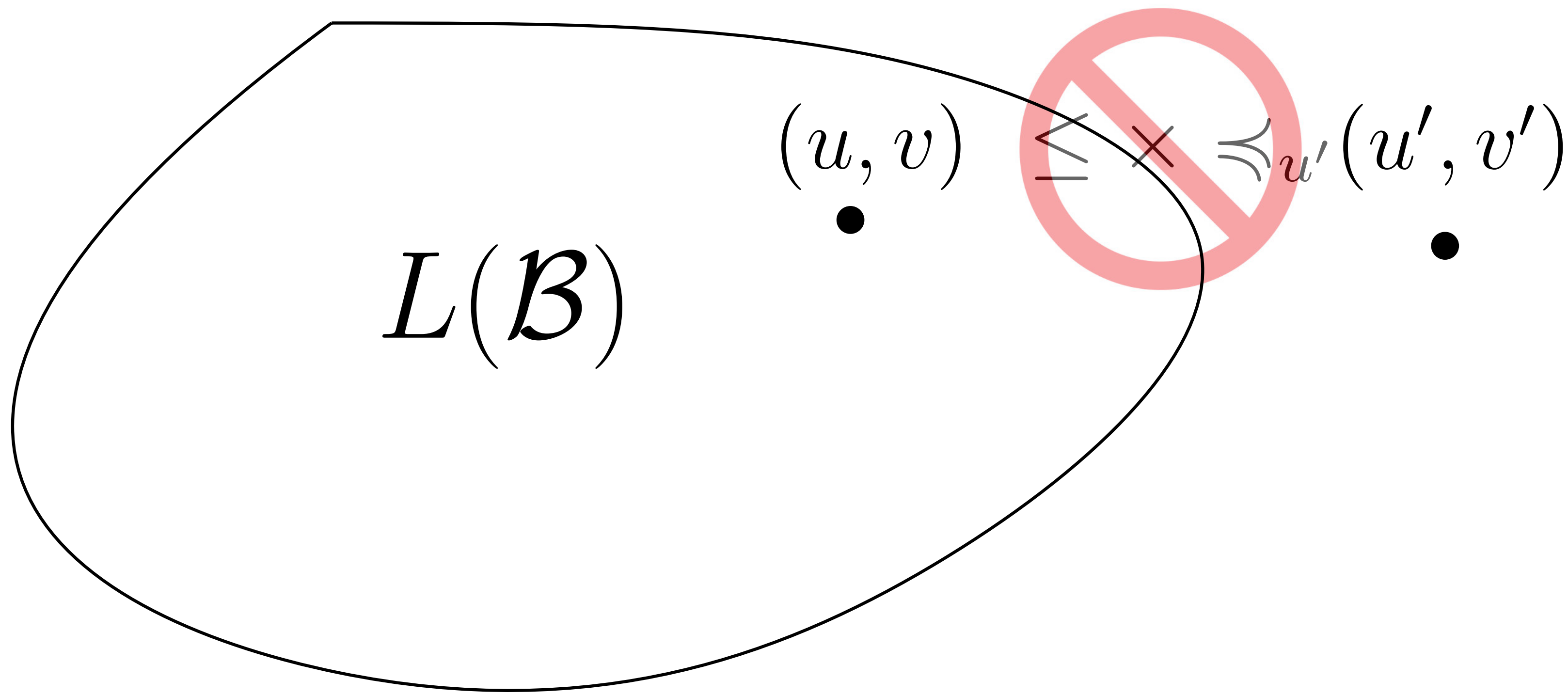
$\forall S \subseteq \Sigma^*$, $\min_{\leq}(S)$ is a finite set and an antichain

Requirements

$$\leq, \preceq$$

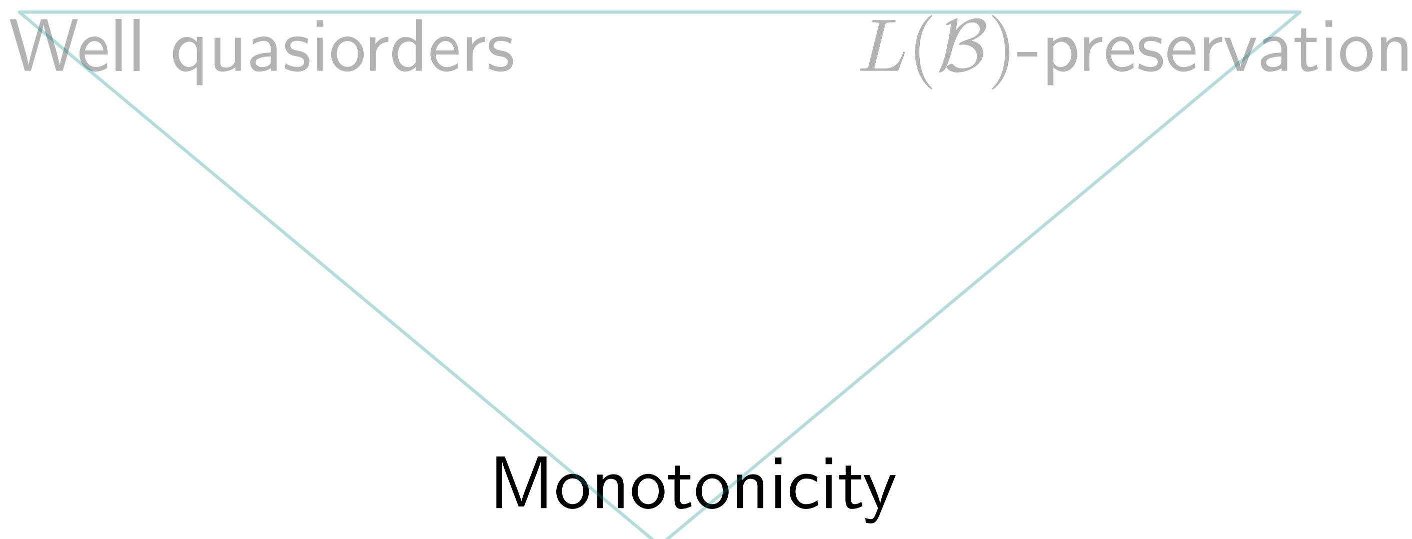


$L(\mathcal{B})$-preservation



Requirements

$$\leq, \preceq$$



Monotonicity

- Specific conditions for the VPL semantics

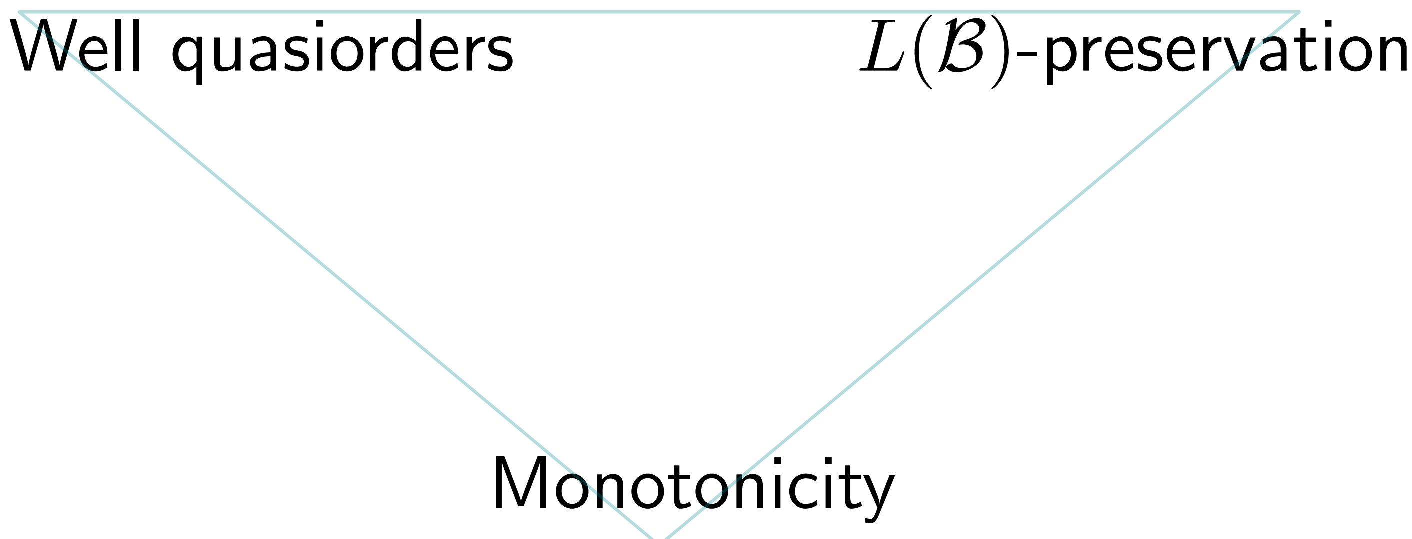
- Iteratively compute S_{finite}

- Safely discard words subsumed by some other words

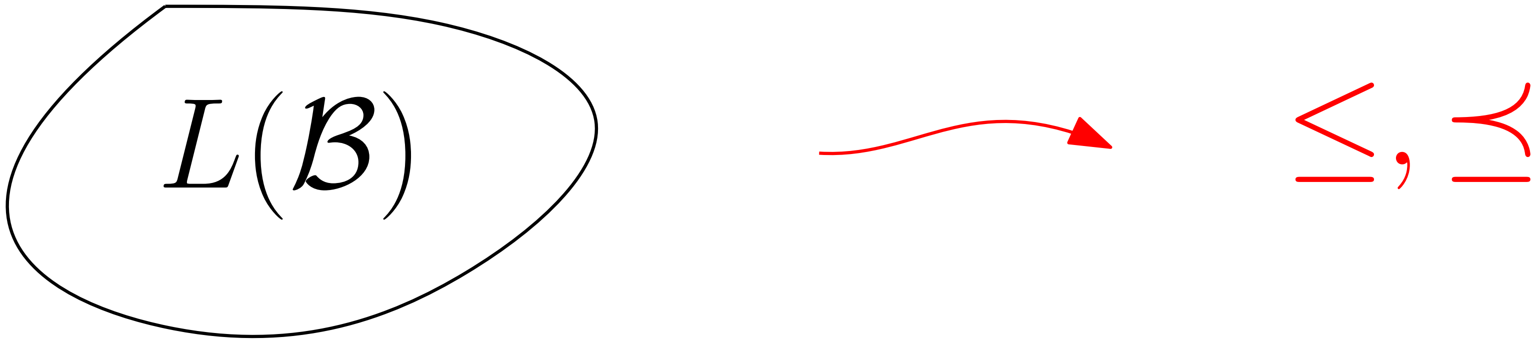
  - keep antichains

Requirements

$$\leq, \preceq$$

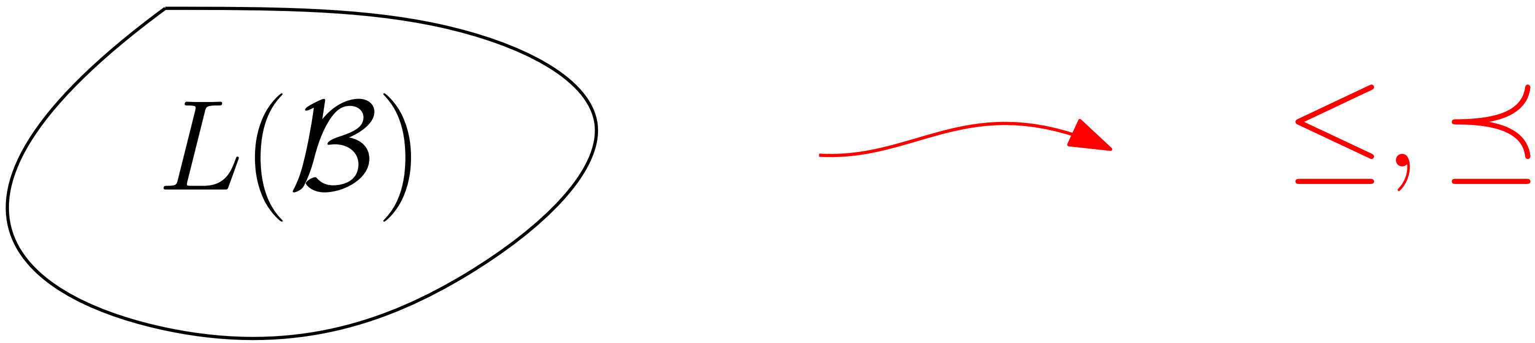


Quasiorders of a VPA



$$u \leq v \iff \text{for every states } p, q \text{ if } (p, \bot) \vdash^{u} (q, w) \text{ then}$$
$$(p, \bot) \vdash^{v} (q, w') \text{ for some } w' \in \Gamma$$

Quasiorders of a VPA



$$u \leq v \iff \text{for every states } p, q \text{ if } (p, \bot) \vdash^{u} (q, w) \text{ then } (p, \bot) \vdash^{v} (q, w') \text{ for some } w' \in \Gamma$$

$$u \preceq v \iff u \leq v + \text{condition w.r.t. accepting states}$$

Inclusion Checking for ω-VPL

Algorithm for $L(\mathcal{A}) \subseteq L(\mathcal{B})$

Data: ω-VPA $\mathcal{A}$, $\mathcal{B}$

Data: Decidable monotonic $L(\mathcal{B})$-preserving wqos $\leq, \preccurlyeq$

Compute ${f_{\mathcal{A}}}^{m}(\emptyset)$ with least m s.t. $\min_{\leq}({f_{\mathcal{A}}}^{m+1}(\emptyset)) \simeq \min_{\leq}({f_{\mathcal{A}}}^{m}(\emptyset))$;

Compute ${r_{\mathcal{A}}}^{m'}(\emptyset)$ with least m' s.t. $\min_{\preccurlyeq}({r_{\mathcal{A}}}^{m'+1}(\emptyset)) \simeq \min_{\preccurlyeq}({r_{\mathcal{A}}}^{m'}(\emptyset))$;

foreach $u \in {f_{\mathcal{A}}}^{m}(\emptyset)$, $v \in {r_{\mathcal{A}}}^{m'}(\emptyset)$ **do**

    if $uv^{\omega} \notin L(\mathcal{B})$ **then return** false;

return true;

Inclusion Checking for ω-VPL

Algorithm for $L(\mathcal{A}) \subseteq L(\mathcal{B})$

fixpoint computation over well-quasiorder domain

Data: ω-VPA $\mathcal{A}$, $\mathcal{B}$

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Compute ${f_\mathcal{A}}^m(\emptyset)$ with least m s.t. $\min_\leq({f_\mathcal{A}}^{m+1}(\emptyset)) \simeq \min_\leq({f_\mathcal{A}}^m(\emptyset))$;

Compute ${r_\mathcal{A}}^{m'}(\emptyset)$ membership check in$_\preccurlyeq({r_\mathcal{A}}^{m'+1}(\emptyset)) \simeq \min_\preccurlyeq({r_\mathcal{A}}^{m'}(\emptyset))$;

foreach $u \in {f_\mathcal{A}}^m(\emptyset)$, $v \in {r_\mathcal{A}}^{m'}(\emptyset)$ **do**

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Experimental Results

Our tool	$L(\mathcal{A}) \cap L(\mathcal{B})^c = \emptyset$
253/281	142/281

https://github.com/hgluka/omegavplinc

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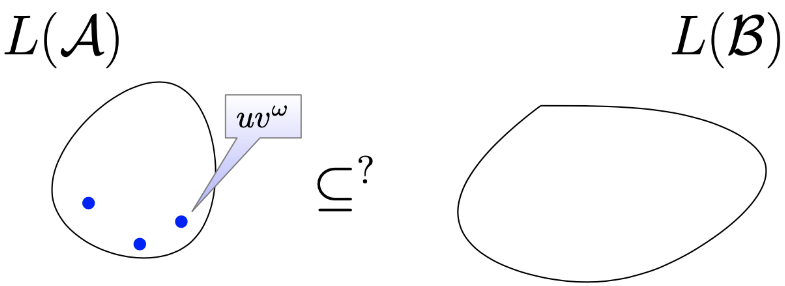
prefix period

$\leq, \preceq$



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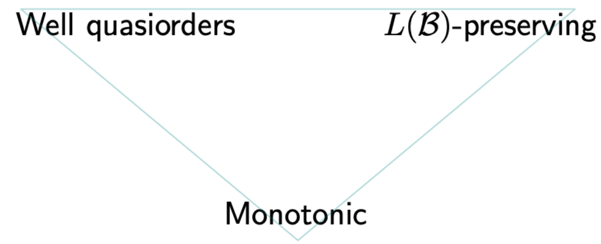
Requirements

$\leq, \preceq$



Algorithm for $L(\mathcal{A}) \subseteq L(\mathcal{B})$

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fixpoint computation over well-quasiorder domain

membership check

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Thank you !