Commutativity in Concurrent Program Verification

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	Marcel Ebbinghaus	(University of Freiburg)

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Example Program

$$\{ \; x = y = i = j = 0 \; \}$$

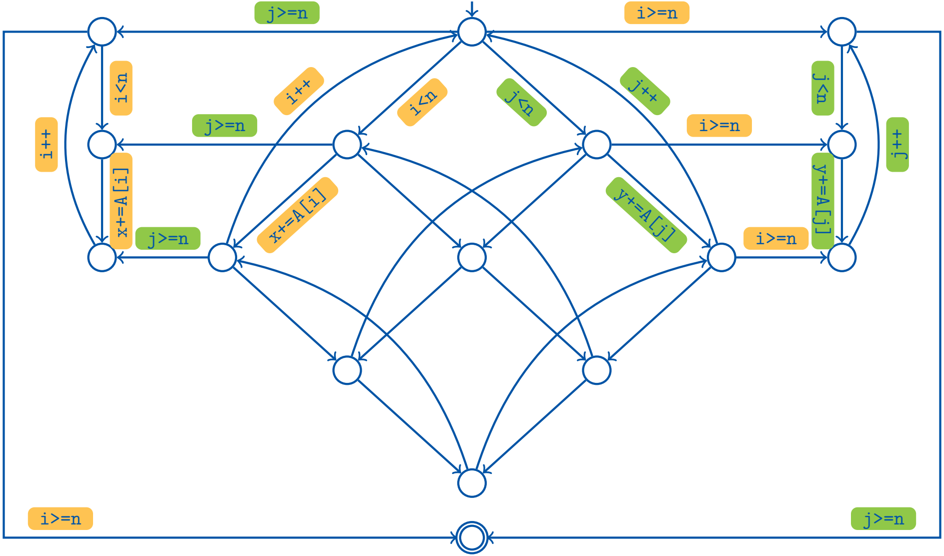
```
while (i < n) {
    x += A[i];
    i++;
}
```

$$\|$$

```
while (j < n) {
    y += A[j];
    j++;
}
```

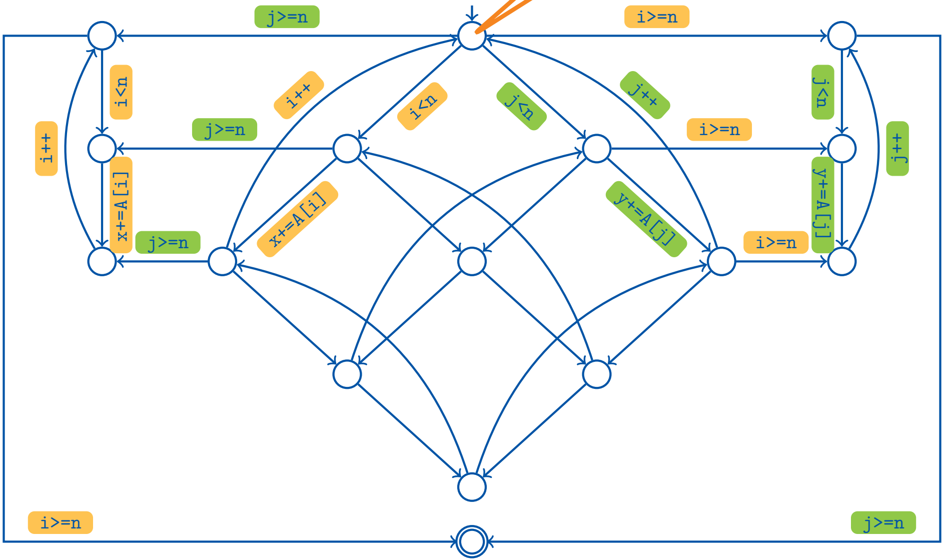
$$\{ \; x = y \; \}$$

Naïve Sequentialization



Naïve Sequentialization

$$x = \sum_{k=0}^{i} A[k] \wedge y = \sum_{k=0}^{j} A[k] \wedge i \leq n \wedge j \leq n$$



Commutativity-Based Equivalence

Many statements **commute**: execution order does not matter

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Reduction

One representative trace for each equivalence class

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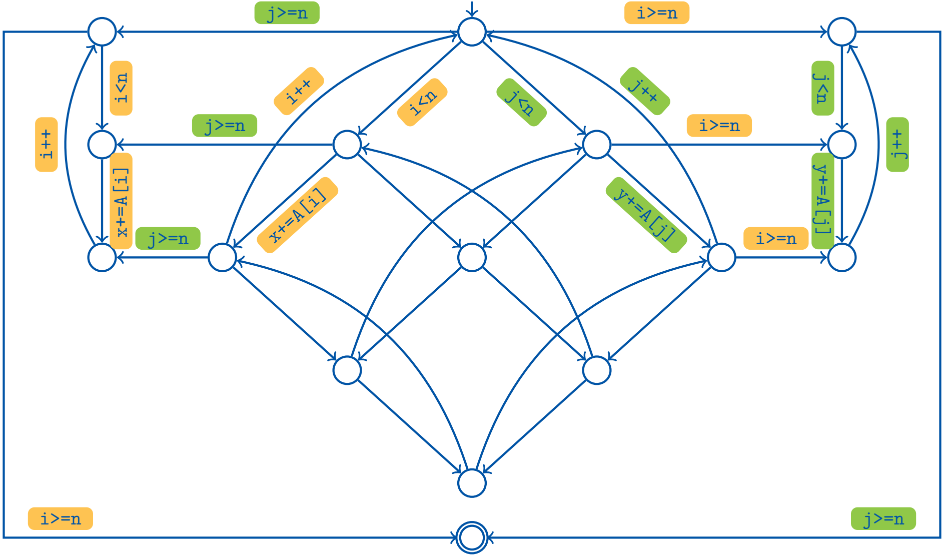

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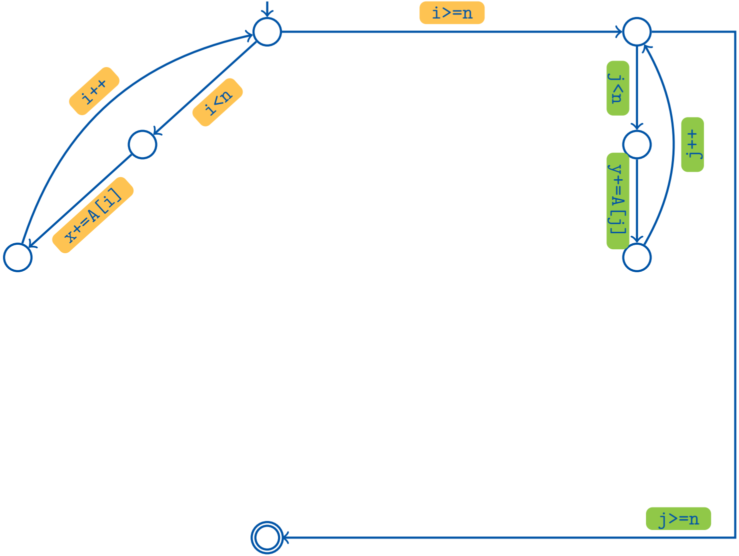

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Iteratively construct **Floyd/Hoare-style proof** of program

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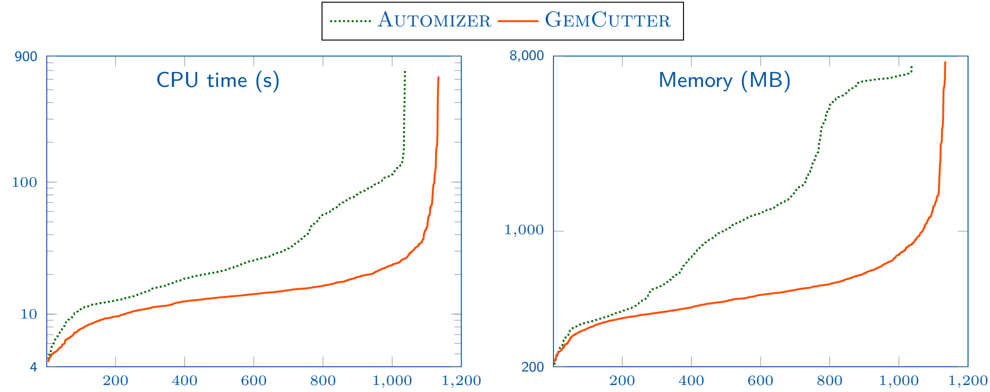
[1] Ebbinghaus. *Tight Integration of Partial Order Reduction into Trace Abstraction Refinement*. BSc Thesis
[2] Klumpp, Dietsch, Heizmann, Schüssele, Ebbinghaus, Farzan and Podelski. *Ultimate GemCutter and the Axes of Generalization - (Competition Contribution)*. TACAS 2022

Evaluation

Implemented in ULTIMATE GEMCUTTER software model checker

Evaluated on SV-COMP'21 benchmarks and benchmarks of WEAVER tool



analyzed **50** more programs using significantly less time ($\approx 50\,\%$), memory ($\approx 27\,\%$), and refinement rounds ($\approx 64\,\%$)

Reductions

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preference order $\preceq$

selects representatives for each equivalence class

Preference Orders

Selecting the right representatives

[3] Farzan, Klumpp and Podelski. *Sound sequentialization for concurrent program verification.* PLDI 2022

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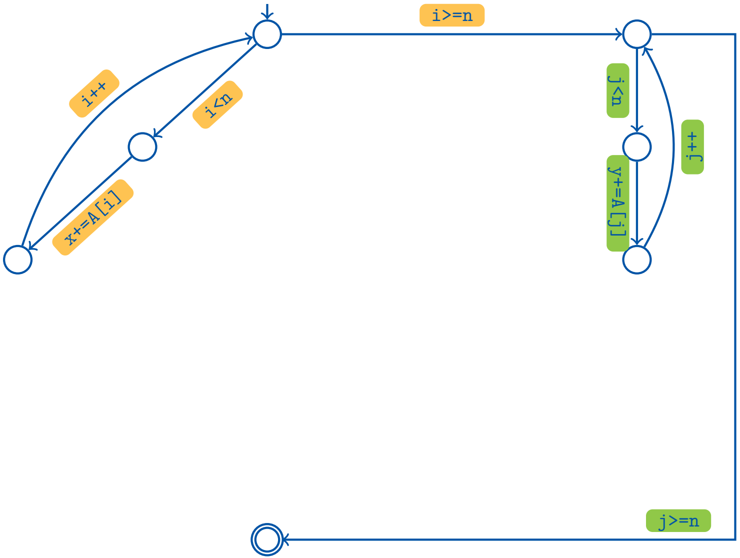

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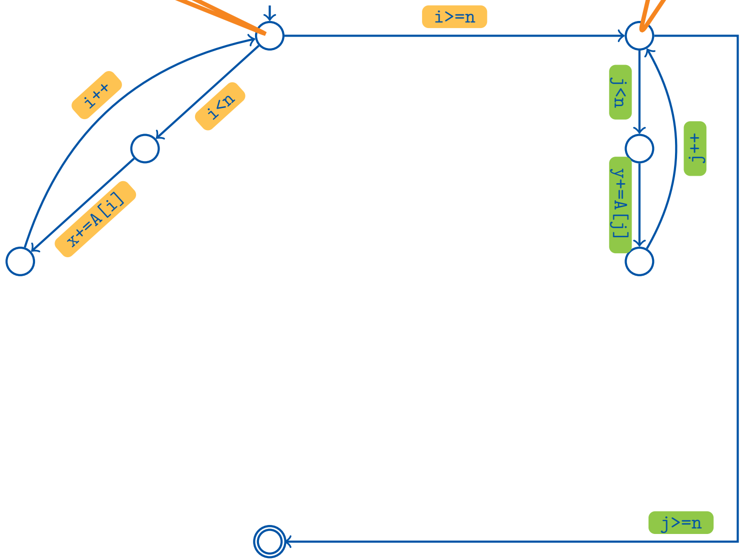

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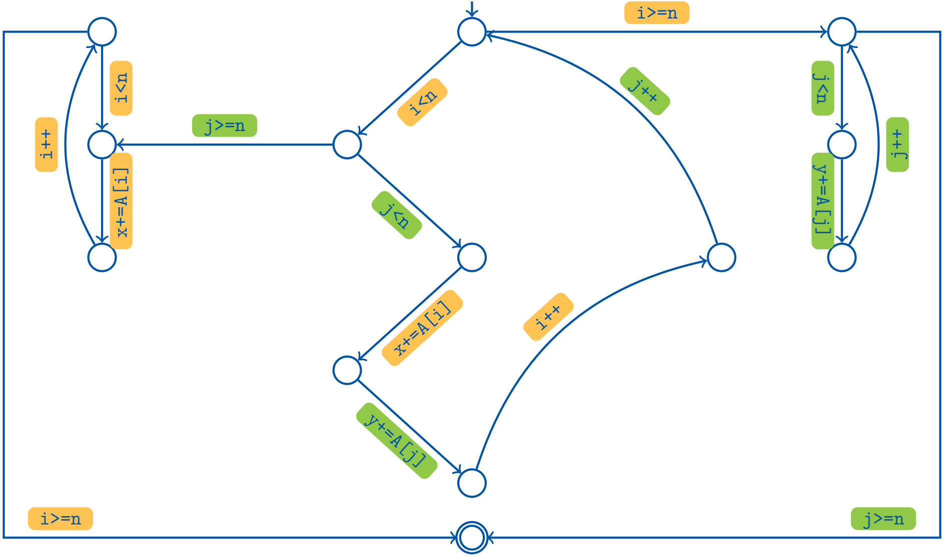

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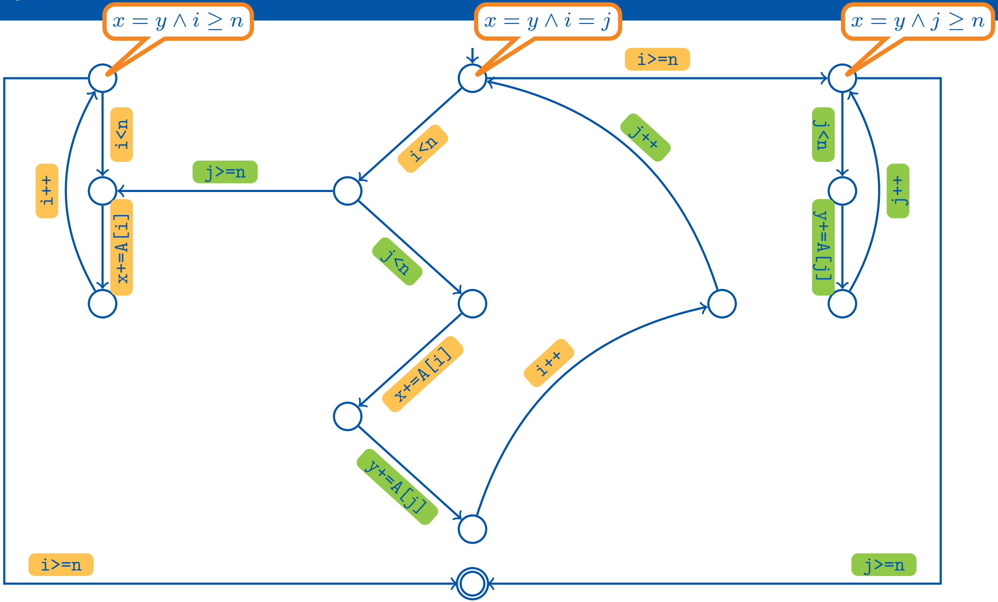

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- independent of commutativity
- same scheme of preference order applies to different programs

Positional Lexicographic Preference Orders

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$\rightarrow$ More on preference orders in **Marcel's talk**

Commutativity Relations

at different abstraction levels

[work in progress; presented at Commute workshop @ PLDI'22]

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Statements st_1 and st_2 **commute**

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($\mathit{st}_1\mathit{st}_2$ behaves exactly like $\mathit{st}_2\mathit{st}_1$)
for all programs and wrt. all properties

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Safe Commutativity

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- commutativity I_C based on (concrete) semantics: safe wrt. all proofs Π
- How to get safe commutativity for a particular proof Π?

Safe Abstraction

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Theorem (Safety)

If α satisfies

- **abstraction:** $[\![st]\!] \subseteq [\![\alpha(st)]\!]$ for all st
- **preservation:** $\{\varphi\}\alpha(st)\{\psi\}$ is valid, for all $\{\varphi\}st\{\psi\} \in \Pi$

then I_α is safe wrt. Π.

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Now: $\alpha_\Pi($ `y:=x+x` $)$ commutes with $\alpha_\Pi($ `x:=0` $)$.

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Solution: combine abstract with concrete commutativity

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$$\underset{\substack{\uparrow \\ \textbf{proven}}}{\tau_1} \quad \overset{\textbf{(1) abstract}}{\sim_{I_\alpha}} \quad \underset{\Rightarrow}{\tau_2} \quad \overset{\textbf{(2) concrete}}{\sim_{I_C}} \quad \underset{\substack{\uparrow \\ \textbf{correct}}}{\tau_3}$$

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Combination through **new proof rule**:

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New **partial order reduction algorithm** for n commutativity relations

Conclusion

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Questions?