PAC-Monitoring

of Fairness Properties

over Markov Chains

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ISTA

Monitoring?

$$\varphi \subseteq \Sigma^\omega$$

Property

$\varphi \subseteq \Sigma^\omega$

Property

$\theta \in \Theta$

Process

$\varphi \subseteq \Sigma^\omega$

Property

$\theta \in \Theta$

Process

$[\![\mathcal{A}]\!] : \Sigma^* \to \{0,1,?\}$

Monitor

$\varphi \subseteq \Sigma^\omega$ $\theta \in \Theta$ $[\![\mathcal{A}]\!]:\Sigma^* \to \{0,1,?\}$

Property | Process | Monitor

θ:

$\mathcal{A}$:

$\varphi \subseteq \Sigma^\omega$

Property

$\theta \in \Theta$

Process

$[\![\mathcal{A}]\!] : \Sigma^* \to \{0,1,?\}$

Monitor

θ: σ_1

$\mathcal{A}$: ?

$\varphi \subseteq \Sigma^\omega$	$\theta \in \Theta$	$[\![\mathcal{A}]\!] : \Sigma^* \to \{0,1,?\}$
Property	Process	Monitor

θ: $\sigma_1 \sigma_2$

$\mathcal{A}$: ? ?

$\varphi \subseteq \Sigma^\omega$

Property

$\theta \in \Theta$

Process

$[\![\mathcal{A}]\!]:\Sigma^* \to \{0,1,?\}$

Monitor

θ: $\sigma_1\sigma_2\sigma_3$

$\mathcal{A}$: ? ? ?

$\varphi \subseteq \Sigma^\omega$ $\theta \in \Theta$ $[\![\mathcal{A}]\!]:\Sigma^* \to \{0,1,?\}$

Property

Process

Monitor

θ: $\sigma_1\sigma_2\sigma_3\sigma_4\sigma_5\sigma_6\sigma_7$

$\mathcal{A}$: ? ? ? ? ? ? ?

$\varphi \subseteq \Sigma^\omega$ $\theta \in \Theta$ $[\![\mathcal{A}]\!]:\Sigma^* \to \{0,1,?\}$

Property

Process

Monitor

θ: $\sigma_1\sigma_2\sigma_3\sigma_4\sigma_5\sigma_6\sigma_7\ldots\ldots\sigma_{t\text{-}1}$

$\mathcal{A}$: ? ? ? ? ? ? ? .... ?

$\varphi \subseteq \Sigma^\omega$

Property

$\theta \in \Theta$

Process

$[\![\mathcal{A}]\!]:\Sigma^* \rightarrow \{0,1,?\}$

Monitor

$\theta\colon\ \sigma_1\sigma_2\sigma_3\sigma_4\sigma_5\sigma_6\sigma_7\ldots\ldots\ldots\sigma_{t-1}\sigma_t$

$\mathcal{A}\colon\ ?\ ?\ ?\ ?\ ?\ ?\ ?\ldots\ldots\ ?\ \ x$

$\varphi \subseteq \Sigma^\omega$

Property

$\theta \in \Theta$

Process

$[\![\mathcal{A}]\!] : \Sigma^* \to \{0,1,?\}$

Monitor

$$\theta: \sigma_1\sigma_2\sigma_3\sigma_4\sigma_5\sigma_6\sigma_7\ldots\ldots\sigma_{t\text{-}1}\sigma_t\Sigma^\omega$$

$w \in \varphi$ or $w \notin \varphi$

$$\Box a \subseteq \Sigma^\omega$$

Property

$\Box a \subseteq \Sigma^\omega$

Property

θ:

$\mathcal{A}$:

$\Box a \subseteq \Sigma^\omega$

Property

θ: aaaaaa

$\mathcal{A}$: ??????

$\Box a \subseteq \Sigma^\omega$

Property

θ: aaaaaaaaaaaaa........aa

$\mathcal{A}$: ???????????????........??

$\Box a \subseteq \Sigma^\omega$

Property

θ: aaaaaaaaaaaaaa........aab

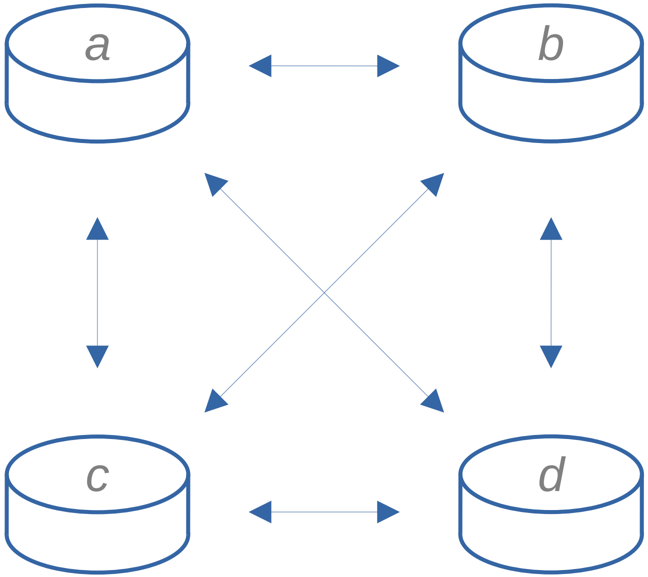
$\mathcal{A}$: ???????????????........??0

$$\Box a \subseteq \Sigma^{\omega}$$

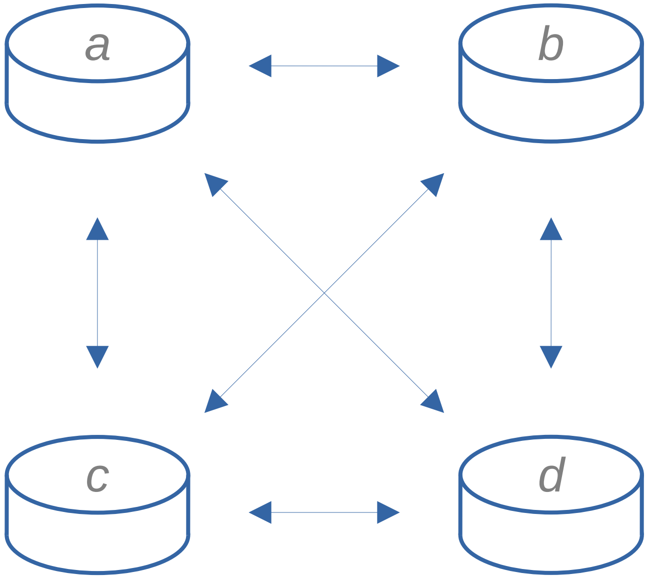
Property

$$\theta: \underbrace{aaaaaaaaaaaaaa\ldots\ldots\ldots aab\Sigma^{\omega}}_{w \notin \Box a}$$

Fairness?

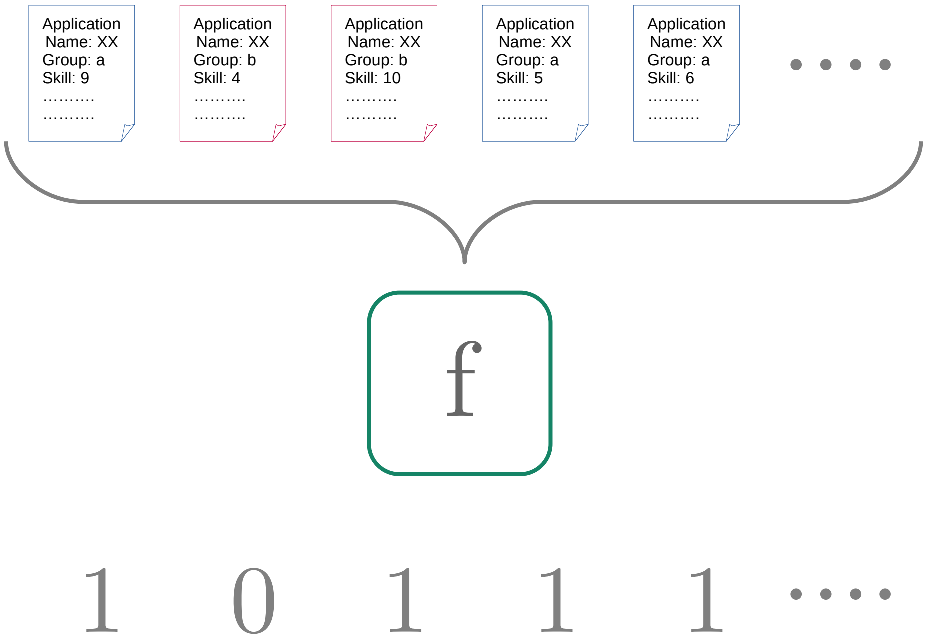

a
b
c
d

Application
Name: XX
Group: a
Skill: 9
Application
Name: XX
Group: b
Skill: 4
Application
Name: XX
Group: b
Skill: 10
Application
Name: XX
Group: a
Skill: 5
Application
Name: XX
Group: a
Skill: 6
f
1 0 1 1 1

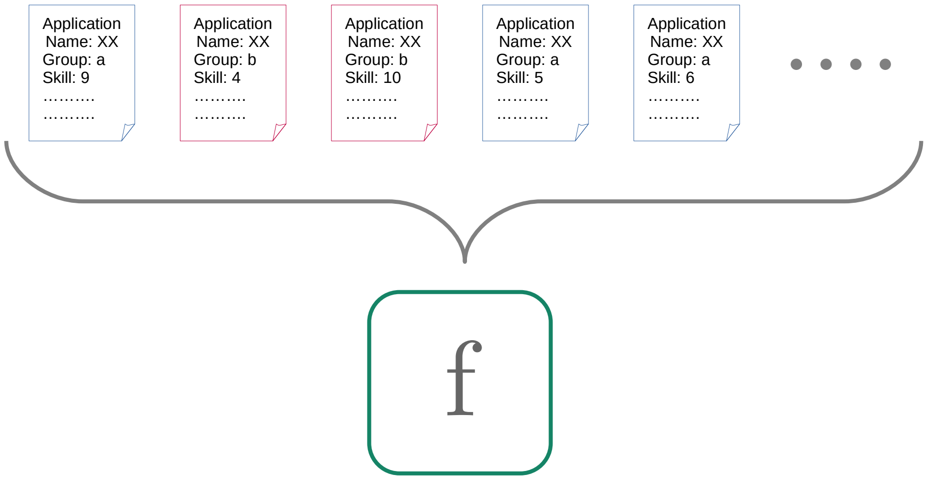


$$|\mathbb{P}(a) - \mathbb{P}(c)| < \alpha$$

$$|\mathbb{P}(b|a) - \mathbb{P}(d|c)| < \alpha$$



$$|\mathbb{P}(b|a) - \mathbb{P}(d|c)| < \alpha$$



1 0 1 1 1 ....

$$|\mathbb{P}(1|a) - \mathbb{P}(1|b)| < \alpha$$

Fairness

Neither a safety nor a liveness.

Fairness

Cannot be monitored!

Why?

Every finite (or infinite) string has a fair and unfair extension.

PAC?

PAC

Probably Approximately Correct

$$\mathbb{P}\left(|\hat{X} - X| < \varepsilon\right) > \delta$$

Probably **A**pproximately **C**orrect

$$\mathbb{P}\left(|\hat{X} - X| < \varepsilon\right) > \delta$$

Probably **A**pproximately **C**orrect

$$\mathbb{P}\left(|\hat{X} - X| < \varepsilon\right) > \delta$$

Probably **A**pproximately **C**orrect

$$\mathbb{P}\left(|\hat{X} - X| < \varepsilon\right) > \delta$$

Probably **A**pproximately **C**orrect

Markov Chains?

$$X_1 \, X_2 \cdots\cdots X_{t\text{-}1} \, X_t \, X_{t+1} \cdots\cdots$$

$$X_1 \; X_2 \; \cdots\cdots \; X_{t\text{-}1} \; X_t \; X_{t+1} \; \cdots\cdots$$

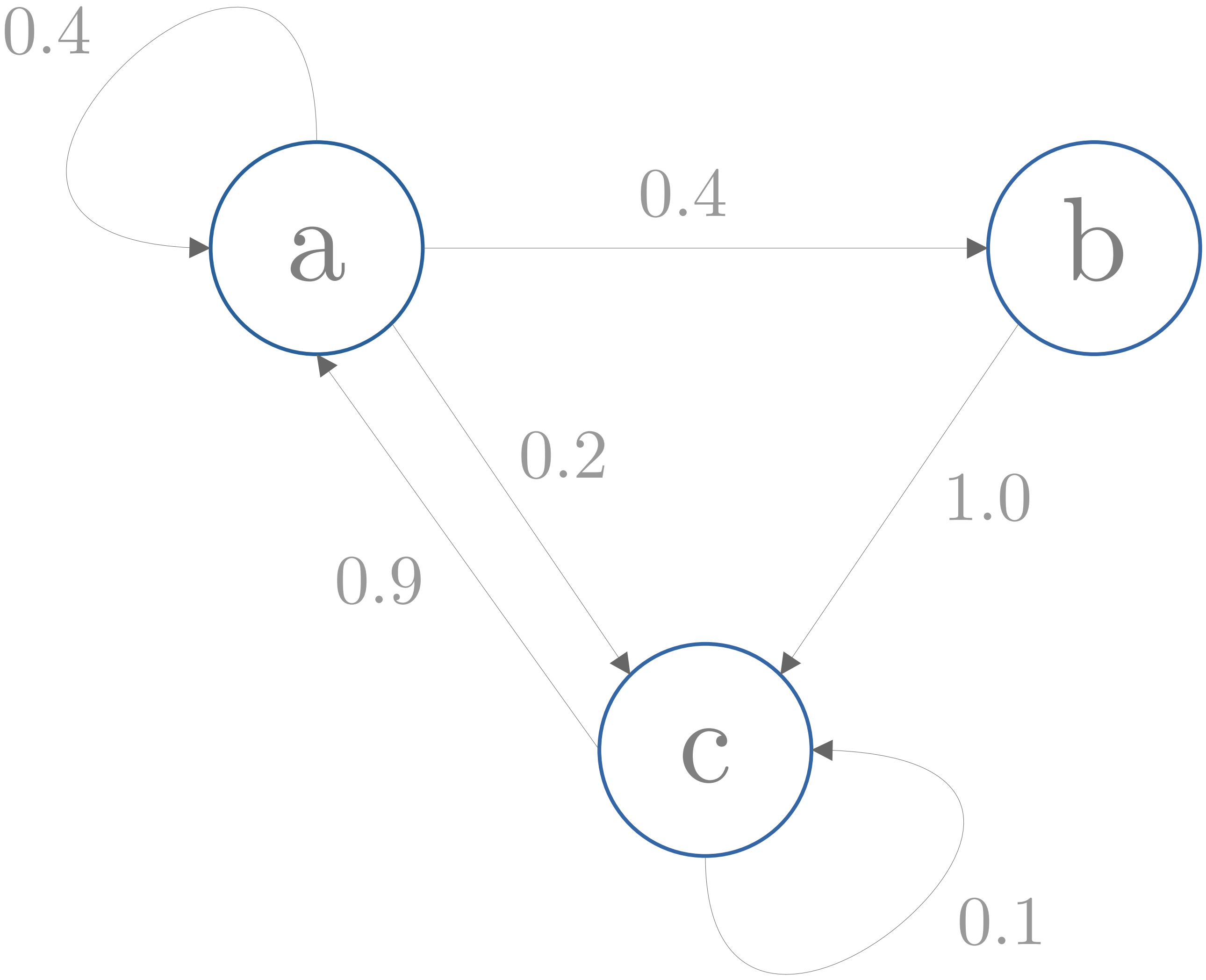
Conditioned on the present

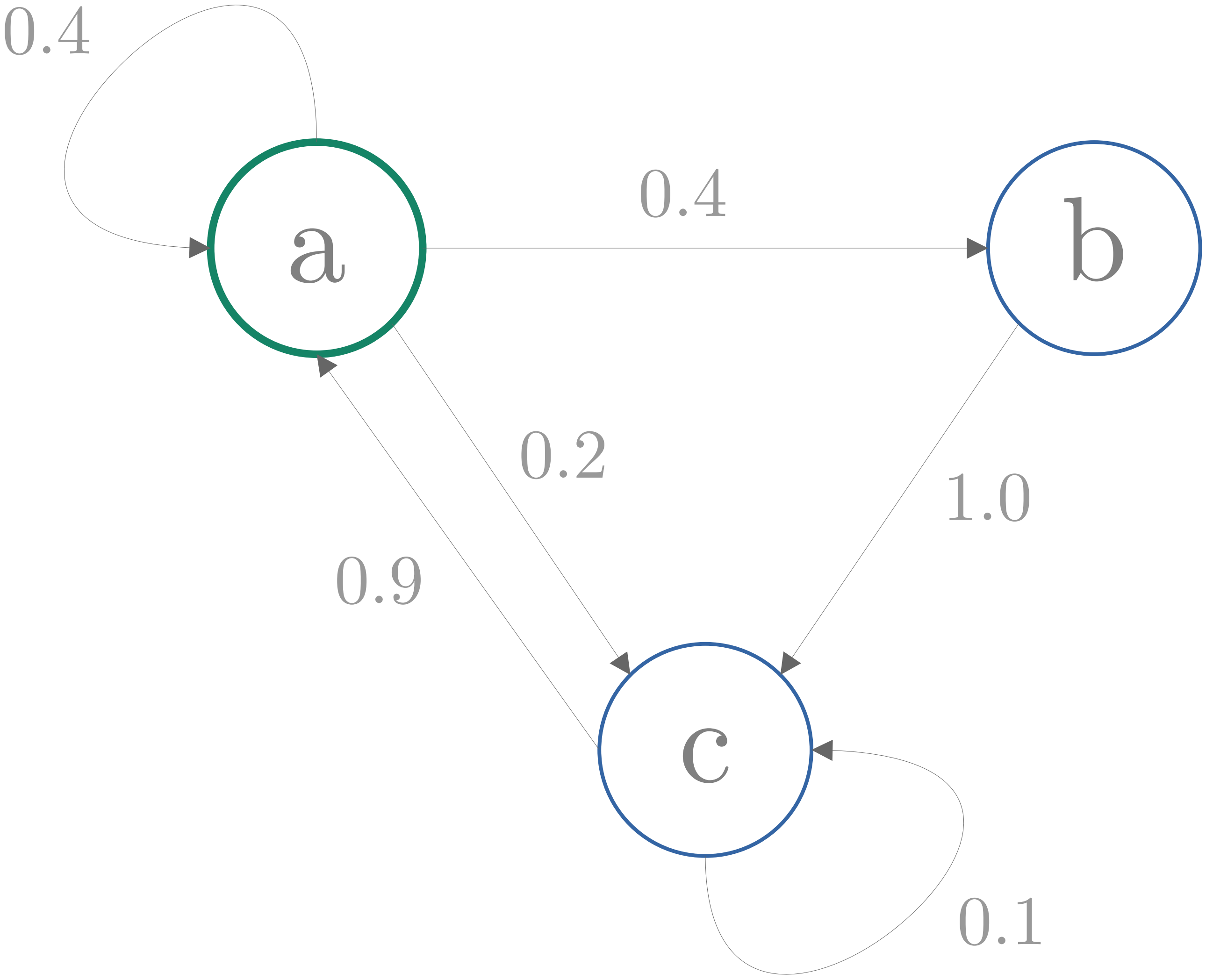
$$X_1 \; X_2 \; \cdots\cdots \; X_{t-1} \; X_t \; X_{t+1} \cdots\cdots$$

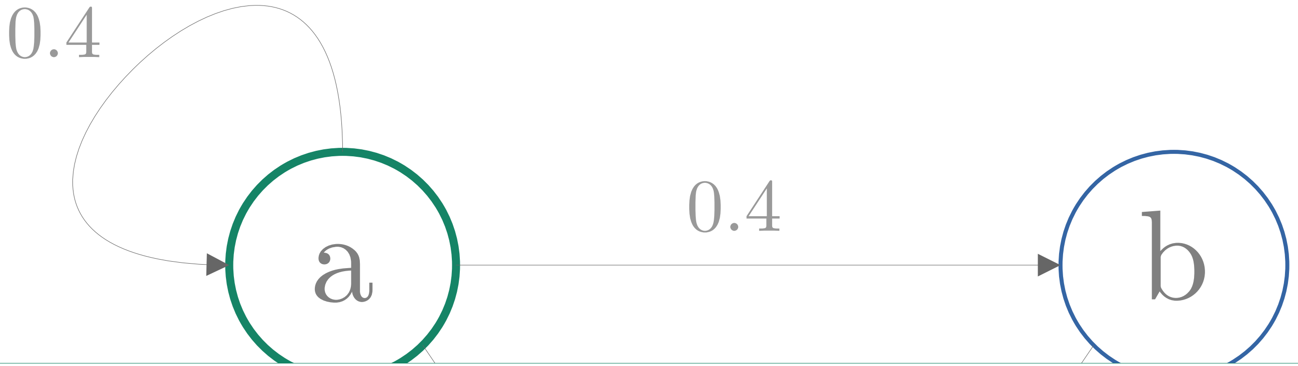
Conditioned on the present
the future does not depend

$$X_1\ X_2\ \cdots\cdots\ X_{t-1}\ X_t\ X_{t+1}\ \cdots\cdots$$

Conditioned on the present the future does not depend on the past.

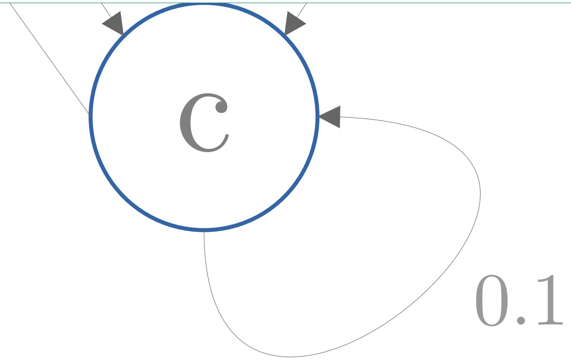

0.4
a
0.4
b
0.2
1.0
0.9
c
0.1

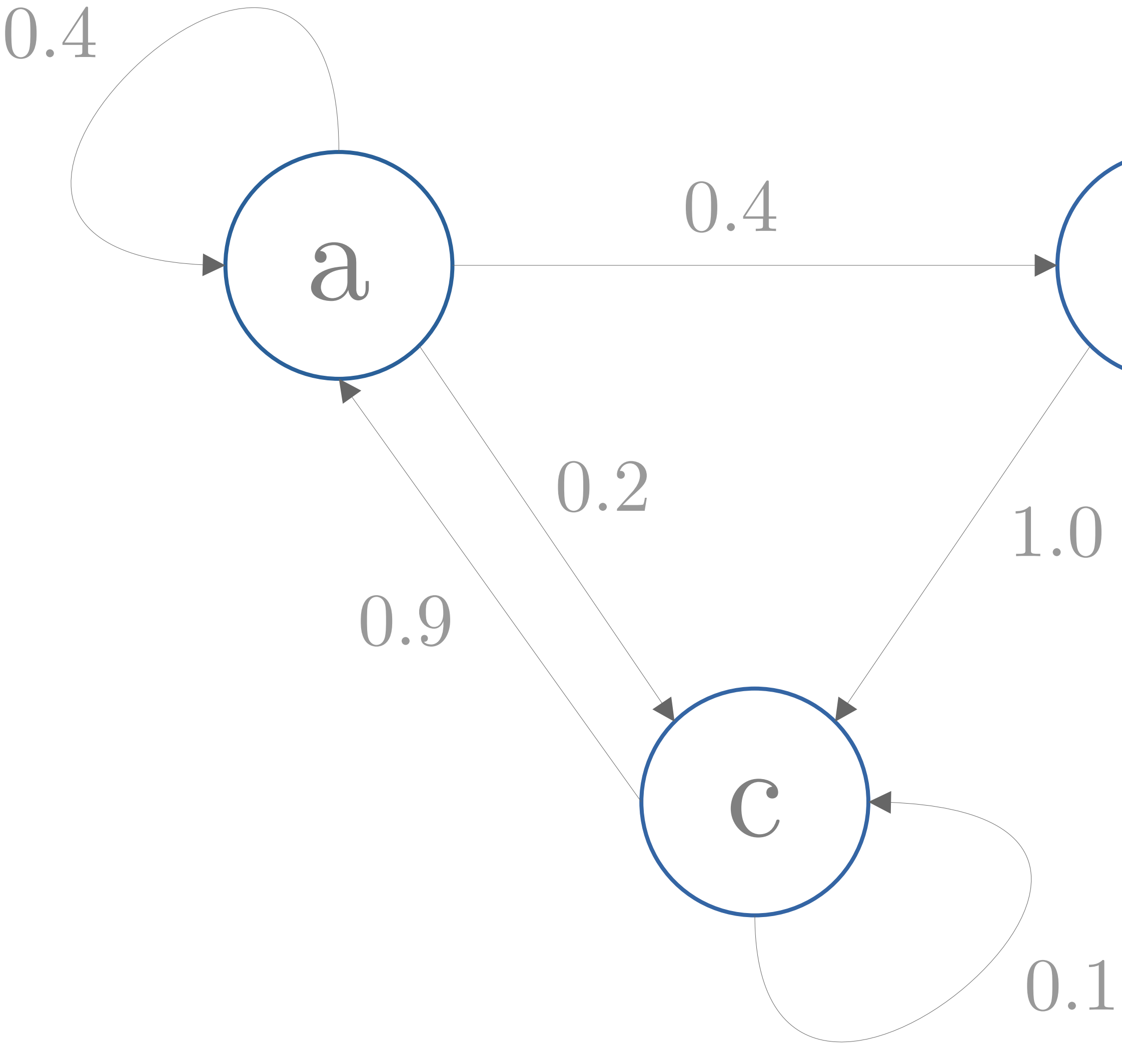

0.4
0.4
a
b
0.2
1.0
0.9
c
0.1



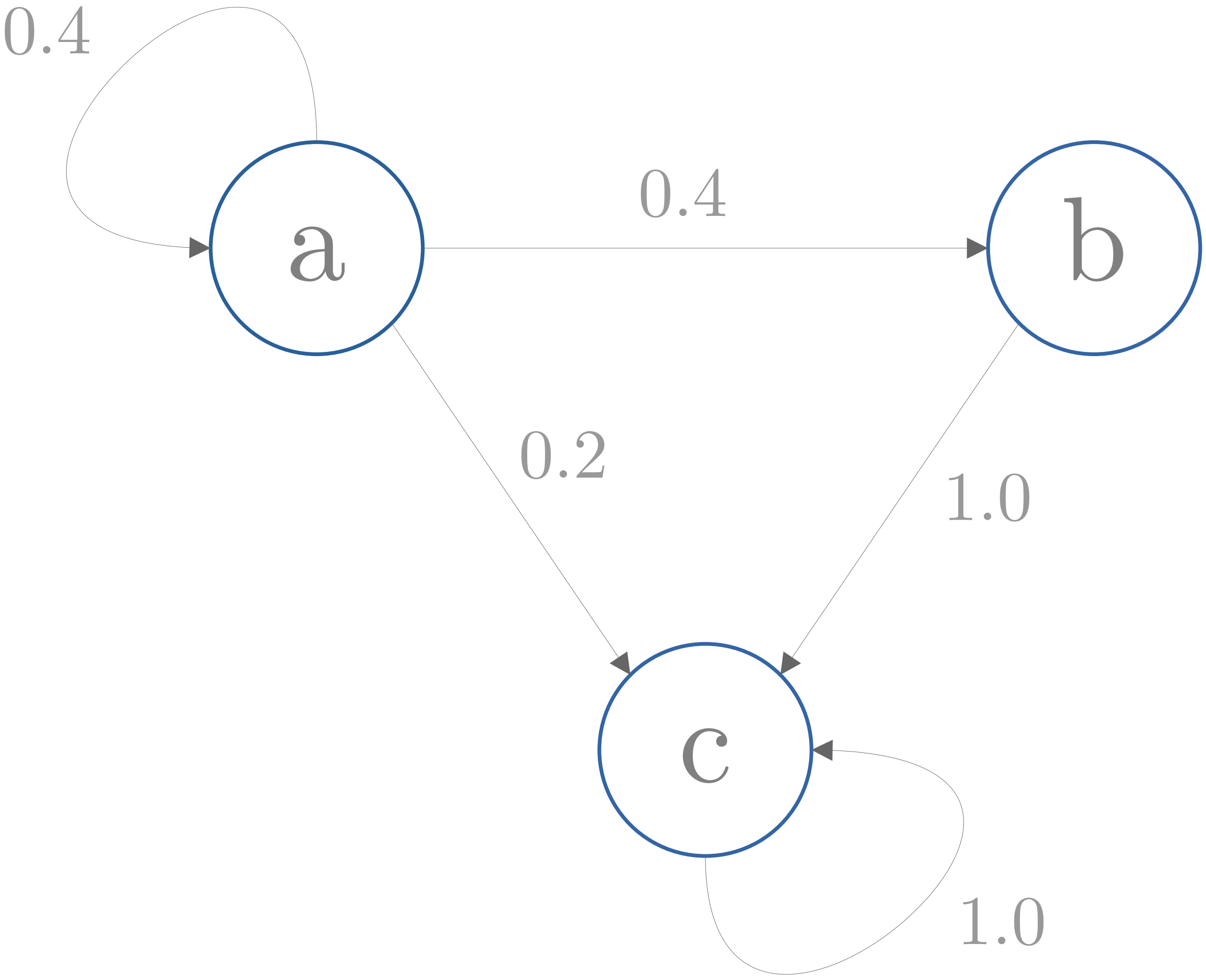
Irreducible:

Probability of reaching each state infinitely often is 1





ISTA


0.4
a
0.4
b
0.2
1.0
c
1.0

PAC-Monitoring

of Fairness Properties

over Markov Chains

What do we want?

$$[\![\varphi]\!]:\Sigma^\omega \to \mathbb{R}$$

Fairness Property

$[\![\varphi]\!] : \Sigma^\omega \to \mathbb{R}$

Fairness Property

$\theta \in \Theta$

Prob. Process

$$[\![\varphi]\!]:\Sigma^\omega\to\mathbb{R}$$

Fairness Property

$$\theta \in \Theta$$

Prob. Process

$$[\![\mathcal{A}]\!]:\Sigma^*\to\mathbb{R}\times[0,1]$$

$$[\![\varphi]\!] : \Sigma^{\omega} \to \mathbb{R}$$

Fairness Property

$$\theta \in \Theta$$

Prob. Process

$$[\![\mathcal{A}]\!] : \Sigma^{*} \to \mathbb{R} \times [0,1]$$

Estimate of φ

$[\![\varphi]\!] : \Sigma^\omega \to \mathbb{R}$

Fairness Property

$\theta \in \Theta$

Prob. Process

$$[\![\mathcal{A}]\!] : \Sigma^* \to \mathbb{R} \times [0,1]$$

Estimate of φ

Confidence

Example

Coin Toss

$[\![\varphi]\!] : \Sigma^\omega \to \mathbb{R}$

Fairness Property

$\theta \in \Theta$

Prob. Process

$[\![\mathcal{A}]\!] : \Sigma^* \to \mathbb{R} \times [0,1]$

Monitor

θ: 0

$\mathcal{A}$: 0 ← Estimate of φ

0 ← Confidence

$[\![\varphi]\!]: \Sigma^{\omega} \to \mathbb{R}$

Fairness Property

$\theta \in \Theta$

Prob. Process

$[\![\mathcal{A}]\!]: \Sigma^{*} \to \mathbb{R} \times [0,1]$

Monitor

θ: 0 1

$\mathcal{A}$: 0 .5
0 0

$[\![\varphi]\!]: \Sigma^\omega \to \mathbb{R}$

Fairness Property

$\theta \in \Theta$

Prob. Process

$[\![\mathcal{A}]\!]: \Sigma^* \to \mathbb{R} \times [0,1]$

Monitor

θ: 0 1 1 0 1 1

$\mathcal{A}$:

0	.5	.66	.5	.6	.66
0	0	0	0	0	.001

$[\![\varphi]\!]:\Sigma^\omega \to \mathbb{R}$

Fairness Property

$\theta \in \Theta$

Prob. Process

$[\![\mathcal{A}]\!]:\Sigma^* \to \mathbb{R}\times[0,1]$

Monitor

θ: 0 1 1 0 1 1 1

$\mathcal{A}$: 0 .5 .66 .5 .6 .66 .71

0 0 0 0 0 .001 .0102

$[\![\varphi]\!]: \Sigma^\omega \to \mathbb{R}$ | $\theta \in \Theta$ | $[\![\mathcal{A}]\!]: \Sigma^* \to \mathbb{R} \times [0,1]$

Fairness Property | Prob. Process | Monitor

θ: 0 1 1 0 1 1 1 ······· 0

$\mathcal{A}$:

0	.5	.66	.5	.6	.66	.71	·······	.502
0	0	0	0	0	.001	.0102	·······	.989

Language.

Syntax

$$\mathcal{L} := \text{Arithmetic}\Big(\mathbb{P}(\tau \mid \sigma)\Big)$$

Syntax

$$\mathcal{L} := \mathrm{Arithmetic}\Big(\mathbb{P}(\tau \mid \sigma)\Big)$$

Semantic

$$[\![\mathbb{P}(\tau \mid \sigma)]\!](w) := \limsup_{t \to \infty} \frac{\#_{\sigma\tau}(w_t)}{\#_{\sigma}(w_t)}$$

Problem Statement.

$$\theta \in \{\text{irreducible MCs}\}$$

$$\theta \in \{\text{irreducible MCs}\}$$

$$u, w \sim \theta$$

finite

infinite

$\theta \in \left\{ \text{irreducible MCs} \right\}$ $\quad u, w \sim \theta$

$$\varphi \in \mathcal{L}$$

$\theta \in \{\text{irreducible MCs}\}$ $u,w \sim \theta$ $\varphi \in \mathcal{L}$

$$[\![\mathcal{A}]\!] : \Sigma^* \to \mathbb{R} \times [0,1]$$

$\theta \in \{\text{irreducible MCs}\}$ $\quad u,w \sim \theta$ $\qquad \varphi \in \mathcal{L}$

$[\![\mathcal{A}]\!] : \Sigma^* \to \mathbb{R} \times [0,1]$

$[\![\mathcal{A}]\!]_1(u)$ estimates $\mathbb{E}([\![\varphi]\!](w))$

$\theta \in \{\text{irreducible MCs}\}$ $u, w \sim \theta$ $\varphi \in \mathcal{L}$

$[\![\mathcal{A}]\!] : \Sigma^* \to \mathbb{R} \times [0,1]$

$[\![\mathcal{A}]\!]_1(u)$ estimates $\mathbb{E}([\![\varphi]\!](w))$

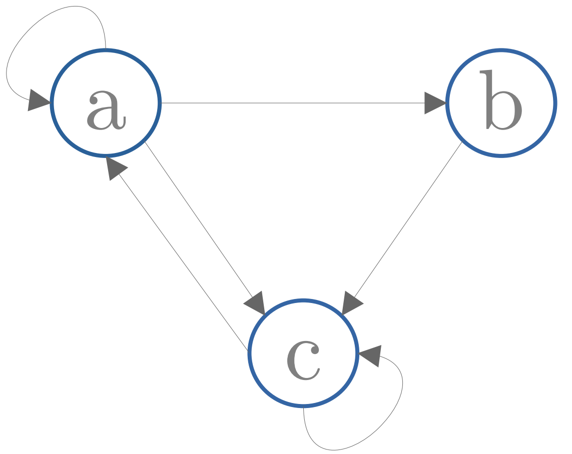
$[\![\mathcal{A}]\!]_2(u)$ confidence in estimate

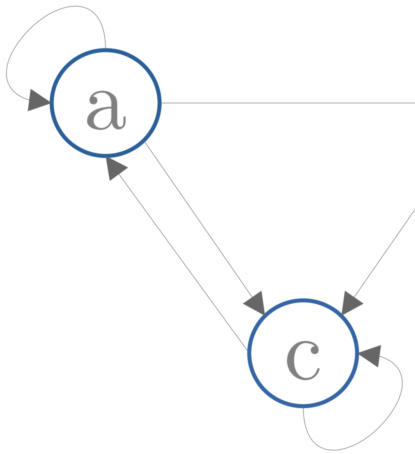
$\theta \in \{\text{irreducible MCs}\}$ $\quad \mathrm{u,w} \sim \theta$ $\quad \varphi \in \mathcal{L}$

$[\![\mathcal{A}]\!] : \Sigma^* \to \mathbb{R} \times [0,1]$

$$\mathbb{P}\left(\left| [\![\mathcal{A}]\!]_1(\mathrm{u}) - \mathbb{E}([\![\varphi]\!](\mathrm{w})) \right| < \varepsilon \right) > [\![\mathcal{A}]\!]_2(\mathrm{u})$$

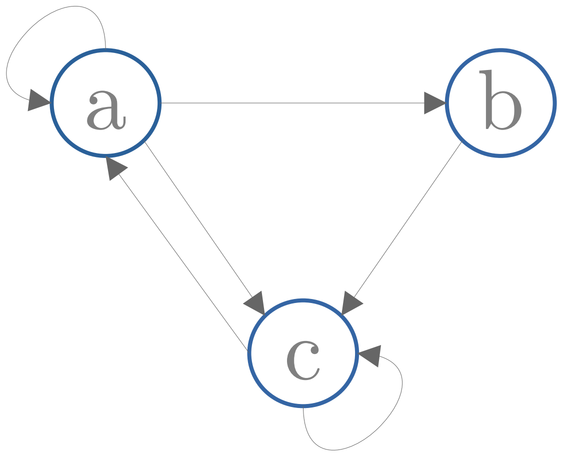
Algorithm.


a
b
c



MP &
irred.

$$\Longrightarrow \quad X_{ab} \; X_{bc} \; \cdots$$



MP & irred.

$$\Longrightarrow \quad X_{ab} \; X_{bc} \; \cdots$$

1 iff b follows a

$$X_{ab}\ X_{bc} \cdots \overset{\text{Combine}}{\Longrightarrow} \quad Y$$

$$X_{ab}\ X_{bc} \cdots \stackrel{\text{Combine}}{\Longrightarrow} \quad Y$$

$$\mathbb{E}(\llbracket \varphi \rrbracket (w)) = \mathbb{E}(Y_i)$$

Empirical estimate

$$\mathrm{Y} \quad \Longrightarrow \quad [\![\mathcal{A}]\!]_1$$

Concentration inequality

$$\mathrm{Y} \quad \Longrightarrow \quad [\![\mathcal{A}]\!]_2$$

Questions?